11. Information networks and knowledge spillovers: simulations in an agent-based model framework

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11.1. INTRODUCTION

Starting from the seminal work of Arrow (1962), new models of endogenous growth theory stress the importance of knowledge spillovers between firms, i.e. the fact that the technological know-how of one firm may positively effect the technological know-how of another firm without full compensation (Romer, 1986; Grossman and Helpman, 1991). In particular, spatial diffusion of knowledge is considered as one of the key factors explaining different regional economic growth. For example, Romer stresses in two seminal papers (Romer, 1992, 1993) that access to the ideas available in leading industrialized countries is critical for many poor countries (or regions).

However, although these new models of endogenous growth theory certainly contribute a great deal to our understanding of the role of knowledge spillovers in economic growth, they still leave some points unsolved.

First, existing studies on knowledge spillovers and regional economic growth focus on knowledge spillovers between a lead and a follower region, while they generally neglect diffusion of knowledge within the follower region. This fact has been nicely pointed out by Rogers (2003).

Second, most existing studies in new economic growth theory consider knowledge spillovers as a black box and do not explicitly model the process of knowledge spillovers between firms. In contrast, many empirical studies emphasize the importance of interaction patterns between firms for knowledge accumulations and economic growth, e.g., communication, information flows, staff exchange, or interlocking directory. Technically, these interaction patterns can be best captured by a network concept. For example, Antonelli (1996) states 'The development of knowledge within industries is strongly influenced by the network structure of relations among firms'. Analogously, Knack and Keefer (1997) stress the importance of social capital, defined as norms and networks that allow collective action, for economic growth.[1]

In general, firms' capacity to absorb new knowledge from external sources can be subdivided into two dimensions: firms' access to external knowledge and firms' capacity to process new external knowledge.

Most studies focus on the latter. For example, one of the first studies focusing on firms' absorption capacity and economic growth is Nelson and Phelps (1966). Nelson and Phelps (1966) assume that firms' capacity to adopt exogenous technological innovation generated in the science sector is determined by human capital of the firm.

Later, other neoclassical studies focus explicitly on knowledge diffusion between firms, but analogously to Nelson and Phelps (1966) understand absorption capacity as the firms' capability to process external technological knowledge. Meanwhile, they implicitly assume perfect access to external knowledge of other firms (Spence, 1984; D'Aspremont and Jacquemin, 1988).

As a critique of simple neoclassical models evolutionary models of innovation diffusion and technological progress have been developed (Dawid, 2006). In contrast to neoclassic, evolutionary economics focuses on an explicit procedural way of representing decision-making rather than relying on abstract optimizing calculus (Dopfer, 2001; Dosi and Winter, 2002; Fagerberg and Verspagen, 2002; Nelson, 1995; Nelson and Winter, 2002). Pioneering work in evolutionary economics in this field apply agent-based models, e.g. Nelson and Winter (1982). Inspired by the work of Nelson and Winter (1982) a large body of literature applying agent based computational economics (ACE) modeling approaches to explain innovation and technological change have been developed, which commonly understand innovation and technological progress as a result of a dynamic process among interacting heterogenous agents. Moreover, this literature highlights in particular the special nature of knowledge as the most important factor for the production of innovation (Dawid, 2006). ACE models contributed significantly to the understanding of the complex process of innovation and technological progress. In particular, Cantner and Pyka (1998) model endogenous strategy to invest in absorbing capacity in the presence of knowledge spillovers in an evolutionary economic framework. However, Cantner and Pyka (1998) also interpret absorption capacity as the firms' ability to process external knowledge, while access to external knowledge is not explicitly considered.

In contrast, research into industrial networks, geography, and innovation clearly underline the importance of network structures of relations among firms for the development of knowledge within industries (Feldman and Audretsch, 1999; Sako, 1999; Paci and Usai, 2000; Antonelli, 1996).

The impact of networks on economic behavior is an innovative and emerging field in economics that also profited tremendously from agent-based computational economics (ACE)-modeling (Wilhite, 2006; Jackson, 2005). However, economic research on networks is still in its infancy and studies on the impact of network structures on innovation diffusion and technological progress do not yet exist. Nevertheless, some notable exceptions exist that focus explicitly on network structures and knowledge spillovers.

An interesting study of the emergence of cooperation networks within a game-theoretical model has been provided by by Goyal and Joshi (2003). In particular, Goyal and Joshi (2003) analyze in a oligopol setting the formation of pair-wise collaboration links among firms, where the set of all links among a fixed set of firms is defined as a collaborative network. An important assumption driving the results of Goyal and Joshi (2003) is how network structures influence firms' profits. Here, Goyal and Joshi (2003) simply assume that marginal costs decrease increasingly with network ties. This specific impact of network structures is only exogenously assumed and is not derived from any detailed mechanism of knowledge transfer between firms. Basically, it corresponds to microeconomic models of knowledge spillovers (Spence, 1984); hence, it would be interesting to analyze the extend to which these assumptions correspond to an explicit model of knowledge diffusion and accumulation in networks.

Another very interesting study considering global network structure is provided by Pyka and Saviotti (2001), who analyze the emergence of collaboration networks in the biotech industry within a complex agent-based model approach. However, while by Pyka and Saviotti (2001), nicely study the emergence of cooperation network structures they do not explicitly analyze how global network structures influence knowledge diffusion and its impact on overall technological progress in the industry.

In this context our chapter offers an agent-based modeling approach to model the process of knowledge spillovers and knowledge accumulation. Technically, we derive a rather simple model that particularly focuses on the role of global network structures as determinants of knowledge diffusion and its implied impact on knowledge accumulation in a network of firms. Applying our simple agent-based model we simulate the impact of different network typologies on knowledge diffusion. In particular, we simulate three different network types, i.e., random, small-world, and scale-free networks, by varying global network structures, which are clustering and centralization.

The rest of the chapter is organized as follows. In the next section we provide a brief background and motivation of our approach. In particular, we relate our approach to relevant existing studies in the literature.

In Section 11.3 we present our formal model. Moreover, we give a brief overview on the network concepts and indicators that we used as we consider these concepts as non-standard to mainstream economists.

In Section 11.4 we discuss central results of our simulation analyses. Moreover, we identify specific framework conditions which amplify or attenuate the impact of network structures on the speed of knowledge accumulation. Finally, in Section 11.5 we conclude and relate our main results to existing theories of knowledge spillovers and regional growth.

11.2. BACKGROUND AND MOTIVATION

Following recent approaches on knowledge spillovers we consider that knowledge spillovers between firms are imperfect. Thus, if we denote the knowledge of firm j by k_j, firm i has only limited access to this knowledge:

$$\theta_{ij} * k_j, \quad \text{with } 0 \le \theta_{ij} \le 1$$

θ_{ij} is the absorption capacity of firm i regarding knowledge of firm j.

The absorption capacity of firm i regarding firm j can be subdivided into two aspects: the capability of firm i to observe technological knowledge of firm j and the capability to process the observed information. We denote π_{ij} as a measurement of the first and λ_{ij} as a measurement for the second capability; then absorption capability results as:

$$\theta_{ij} = \lambda_{ij}\pi_{ij}$$

We cover the first aspect via an information network $\Pi = \left[\pi_{ij}\right]$, where π_{ij} denotes the information exchange frequency between firm i and j.

Thus, firm i can only observe the knowledge of firm j if i and j have established an information network tie where π_{ij} denotes the strength of this tie. Following quantitative network theory as well as recent approaches on networks in economics, e.g., Jackson (2005), conclude that establishing and maintaining a network tie is costly, i.e., requires resources. Accordingly, firms do not establish information ties with all other firms, but only with a share of firms. Empirical network studies show that information networks among firms are sparse, e.g., the average number of information ties observed for a firm rather exceeds 10 (Henning and et al., 2008). This is very low for a larger set of firms, compared to the maximal possible number of ties.

What is the impact of the structure of information networks on the absorption capacity of firms and thus on knowledge spillovers and accumulation? The answer to these questions crucially depends on how the process of knowledge transfers and processing is designed.

These questions also highlight a common observation, that knowledge spillovers between firms cannot be the only source of knowledge. Accordingly, studies generally consider an external source of knowledge. Let k_o denote the external source of knowledge. Following the seminal paper of Nelson and Phelps (1966) we interpret this external source of knowledge as the knowledge sector, e.g., universities, research labs, etc. Accordingly, k_o is the maximal stock of knowledge or techniques available to firms, while k_i denotes the best practice technology realized by firm i.[2]

Analogous to firm spillovers, the capacity to absorb knowledge of the external source is also limited. For example, Nelson and Phelps (1966) analyzed firms' limited capacities to absorb knowledge of the science sector, while other scholars study firms' limited capacities to generate new knowledge via learning-by-doing or R&D-activities (see Cantner and Pyka, 1998). In general, we define:

$$\theta_{io} * k_o, \quad \text{with } 0 \le \theta_{io} \le 1$$

Neoclassical studies such as Spence (1984) focus only on firms' limited processing capacity, λ, assuming perfect access, $\pi_{ij} = 1 \ \forall i, j$.

Moreover, as Spence (1984) presumes that all firms have the same limited processing capacity, it follows:

$$\theta_{ij} = \lambda = \theta$$

Furthermore, as Spence (1984) implicitly assumes that only newly generated knowledge (denoted by Δk_{io} in the following) can be transmitted among firms. Knowledge is generated proportionally to resources allocated to R&D activities. Let m_i denote the amount of resources firm i allocated to R&D, then as Spence (1984) assumes: $\Delta k_{io} = m_i$. Now, firms have access to newly generated knowledge of other firms, where θ denotes the share of new knowledge of firm i that is available to all other firms j (this follows since $\pi_{ij} = 1 \ \forall j$). Thus it holds:

$$\Delta k_i = \Delta k_{io} + \theta \sum_j \Delta k_{jo} = m_i + \theta \sum_j m_j$$

Accordingly, the only network impact on technological diffusion that as Spence (1984) identifies is a simple size effect, e.g., the larger the total set of firms, the larger is c.p. the knowledge generation induced by knowledge spillovers between firms. However, as Spence (1984) did not intend to analyze network effects. Instead he focused on the decision to invest in R&D

(m_i) in the presence of spillover effects. Obviously, in the presence of knowledge spillovers, an increase of firms' absorption capacity of external knowledge implies external effects, as this also increases the knowledge of other firms, i.e., it follows directly $d\Delta k_j / dm_i = \theta \geq 0.$ Spence (1984) nicely demonstrated that firms' incentives to invest in their own R&D crucially depends on market structures.

Analogously to Spence (1984), Cantner and Pyka (1998) also analyze firms' incentives to invest in R&D in comparison to investing in absorption capacity. However, while Cantner and Pyka (1998) in contrast to Spence (1984) undertake their analyses in an evolutionary economic framework, their study also focuses on firms' restricted capacity to process knowledge, i.e., a restriction of $\lambda,$ while they do not consider network effects, this means firms' restricted access to knowledge.

A simple way to introduce imperfect access in the model set-up of Spence (1984) or Cantner and Pyka (1998) would be to assume that firms establish network ties only to a subset of other firms, e.g., we define the neighbourhood of firm $i,$ $N_i:$ $N_i = \{j \in N | \pi_{ij} > 0\}.$

Moreover, considering network structures as endogenous variables, it also follows quite plainly that firms have incentives to invest in their networks, where $\theta \cdot m_k$ would be the additional knowledge spillovers resulting from a tie to a firm k.

This is basically the approach of Goyal and Joshi (2003), who studied firms' incentives to establish cooperation ties with other firms, so that cooperation among firms induces knowledge spillovers and thus has an impact on firms' profits. However, of Goyal and Joshi (2003), did not explicitly analyze how network structures influence knowledge spillovers and thus firm profits. Instead, they simply assumed that the firms' marginal production costs decrease with the number of ties.

Based on this ad hoc assumption, they could derive stable and efficient network structures. Of course, firms preferences over network ties result from the anticipated impact of those ties on firm profits. It follows quite plainly that the impact of an additional network tie on firms' profit can be quite different depending on the concrete process of knowledge diffusion. In particular, we demonstrate that central assumptions of Goyal and Joshi (2003), regarding the impact of network structures on firms' profit would have to be revised in the framework of our model.

A very interesting study considering global network structures is provided by by Pyka and Saviotti (2001), who analyze the emergence of collaboration networks in the biotech industry within a complex agent-based model approach. In contrast to to Goyal and Joshi (2003), Pyka and Saviotti (2001) apply an evolutionary economic approach that explicitly takes into account the fundamental uncertainty and complexity characterizing firms' decisions to establish networks ties. In particular, the evolutionary economic approach of Pyka and Saviotti (2001) emphasizes – in contrast to Goyal and Joshi (2003) – that strategic investment in network ties is often a very complex process characterized by fundamental uncertainty. Accordingly, often it is not straightforward to derive rational strategies regarding how firms should invest in network ties, and therefore firms apply more simple – though certainly not always optimal – heuristic rules of thumb.

However, while Pyka and Saviotti (2001) nicely study the emergence of cooperation network structures they do not explicitly analyze how global network structures influence knowledge diffusion and its impact on overall technological progress in the industry.

Overall, it is still fair to conclude that although empirical studies have clearly recognized the importance of established informal and formal interfirm network structures for knowledge accumulation and its role in innovation processes, only a few (though very interesting) theoretical studies on the impact of networks on knowledge spillovers exist. Accordingly, many issues have not been fully addressed. For example, the impact of information network structures on knowledge diffusion between firms and its impact on the speed of knowledge accumulation has not been comprehensively analyzed.

That gap is addressed in this paper. Therefore, in the next subsection we derive a simple agent-based model to study how knowledge diffuses and is accumulated in a set of firms, explicitly including the structure of information network channels in which knowledge flows between firms.

11.3. THE MODEL

11.3.1. Basic Assumptions

The aim is to model the impact of global and local network structures on aggregate knowledge accumulation within a set of firm. We consider a set of business firms N, with $i = 1,...,n$ denoting an individual firm.

Moreover, accumulation of knowledge involves several mechanisms, e.g., in-house R&D, informal transfer of knowledge between companies (spillovers), or learning-by-doing. In this paper we focus on the diffusion of knowledge between firms rather than on the process of generation of new knowledge by firms. Accordingly, we assume that new technological knowledge is constantly generated outside of the firm network, e.g. in an external science sector or in an external leading region. Firms can directly absorb external knowledge generated outside the network or absorb knowledge from other firms in the network.[3]

Modeling knowledge accumulation, we incorporate fundamental results of previous studies on technological spillovers. First, the firms' capacity to absorb external knowledge is generally limited and can be subdivided into two aspects: the firms' ability to observe external knowledge and their capacity to process observed knowledge, respectively.

Regarding the latter aspect, we follow Cantner and Pyka (1998) and assume that the firms' ability to process received information crucially depends on their current stock of knowledge. As will be elaborated upon below, we formally incorporate the impact of the firms' current knowledge on knowledge accumulation. We do this by assuming that new technological knowledge can only be processed if the firms' current knowledge is already above a specific threshold. Moreover, we assume that information processing is a stochastic process in which firms' limited ability to process external knowledge is measured as the expected amount of knowledge a firm is able to process.

An essential characteristic highlighted in previous studies (Nelson and Phelps, 1966; Cantner and Pyka, 1998) is that knowledge from other firms in the network is generally easier to process than new knowledge from an external source, e.g., the science sector or a leading region or generating it via learning-by-doing processes. Basically, this feature makes knowledge accumulation within a network of firms a collective process (see Cantner and Pyka, 1998). In our model we take this essential characteristic into account by assuming that c.p. firms observe a much higher capacity to process knowledge from other firms in their network when compared to processing knowledge from an external source outside the network.

Moreover, previous studies suggest that firms can invest in their capacity to process knowledge Cantner and Pyka (1998).[4] However, at this stage we try to keep our model simple and therefore we assume that the firms' capacity to process knowledge from other firms is constant and exogenously determined.

Regarding the first aspect – firms capacity to observe knowledge – we have already established that diffusion of knowledge between firms is not trivial, this means it requires the existence of an established information network channel between firms (Cohen and Levinthal, 1989; Rosenberg, 1990). Transmission of new technological knowledge between firms can only occur if staff members of two firms communicate with each other. Obviously, communication between firms requires specific opportunities to interact, e.g., doing business or meeting within a business organization or even meeting at social events such as dinner parties, playing golf, etc. Of course, depending on the specific business, organizational and social opportunities for interactions vary significantly across firms. Accordingly, the frequency of information transmission varies significantly across pairs of firms. As will be shown in detail below, the firms' opportunities to communicate can be captured by defining firm specific information network ties, where the structure of information networks has a crucial impact on knowledge accumulation.

To facilitate our analyses we focus our studies on a binary information network Z, where the latter is defined as a graph of N^2, e.g., the information network is a subset of pairs $(i,j)\in N^2$, where $i,j\in N$ are usually called vertices and a pair (i,j) is called an edge. Let $z_{ij}=1$ indicate a tie between firms i and j implying that firms i and firm j have an established information channel. Then the binary information network Z can be defined as follows:

$$Z=\left\{(i,j)\in N^2 \middle| z_{ij}=1\right\} \tag{11.1}$$

Information flows, π_{ij}, between firm i and j can only occur in established information channels. Thus, let π denote the common conditional probability that information flows within an established information channel: then a valued information network, Π, results from our binary information network Z via multiplying with the scalar π: $\Pi=\pi Z$.

In detail, networks can have different structures and different graphs have different topologies. Quantitative network theory developed a set of local and global network indicators to describe specific network structures and typologies (Wasserman and Faust, 1994).

Before we present our agent-based model (ABM) and simulation analyses we will briefly define relevant global and local network indicators. Although these network indicators are described in standard textbooks of quantitative network analysis (Wasserman and Faust, 1994), they are not standard in the economic literature.

11.3.1.1. Network indicators

Given the fact that we are mainly interested in the impact of social network structures on information and innovation diffusion beyond global network size, n, the following global and local indicators are of interest:

Local network size

If we denote the number of ties an individual firm i forms by z_i, then local network size, $\overline{z}$, is defined as the average number of ties of firms:

$$\overline{z}=\frac{\sum_{i\in N} z_i}{n} \tag{11.2}$$

Network density

Network density is defined as the number of realized ties divided by the number of theoretically possible ties.

$$\phi = \frac{\sum_{i \in N} z_i}{n^2} = \frac{\overline{z}}{n} \tag{11.3}$$

To simulate density we keep total network size n constant and simulate an average number of network ties, $\overline{z}$.

Global centralization

Global centralization of a network measures the difference in individual network ties (degrees). Accordingly, the larger the variance of individual degrees of actors, the larger is c.p. the centralization. Let σ^2 denote the variance of degrees in a network, then it holds:

$$\sigma^2 = \frac{1}{n}\sum_{i \in N}\left(z_i - \overline{z}\right)^2 \tag{11.4}$$

Thus, by definition, centralization is measured by the variance or standard deviation of network degrees, σ^2 or σ, respectively.

Clustering/transitivity

Network clustering or transitivity is defined as the average density of the actor's neighborhood network. A neighborhood network N_i of an actor $i \in N$ is defined as the subset of actors $j \in N$ which have a tie to i:

$$N_i = \left\{ j \in N \middle| z_{ij} = 1 \right\} \tag{11.5}$$

Now, given z_i neighbors of an actor i, the density of the neighborhood network N_i is defined as:

$$\gamma_i = \frac{\sum_{k \in N_i} \sum_{j \in N_i} z_{kj}}{z_i(z_i - 1)} \tag{11.6}$$

Finally, the global clustering is defined as the average density of all neighborhood networks:

$$\gamma = \frac{1}{n}\sum_{i \in N}\gamma_i \tag{11.7}$$

Characteristic path length
We define g_{ij} as the minimum path length connecting an actor i with an actor j. To define g_{ij} formally we define Z_{ij}^r as the ij-component of the matrix Z^r, with $Z = [Z_{ij}]$. Then g_{ij} is defined as follows:

$$g_{ij} = \underset{r}{Min}\left\{ r \middle| Z_{ij}^r > 0 \right\} \tag{11.8}$$

Further, we define g_i as the mean of all g_{ij}. Then the characteristic path length (L) is defined as the median of all means g_i. A problem arises if a network is not a strong component, i.e. some actors are isolated. The path length to isolated actors becomes infinitely. In this case the characteristic path length can not be calculated. A solution to this problem would be to calculate the characteristic path length only for the strong component of the network, i.e. the largest subset of actors that are connected with each other via a finite path.

11.3.1.2. Random, small-world and scale-free networks

In the emerging literature on 'networks and economics' different types of networks characterized by specific network indicators have become very popular, i.e., random networks, small-world networks, and scale-free networks (Jackson, 2005). Therefore, we briefly define these specific network types based on characteristic values for global network indicators. Hence it is helpful to identify the range of network types and their implications for measuring performance by network indicators.

Random graphs
A random graph is a graph in which the edges between the vertices are generated randomly. $Z(n,p)$ with a natural number $n \geq 1$ and a probability p denotes the class of all graphs where, for every tuple (i,j) of vertices, the probability p determines if they are connected, i.e., $z_{ij} = 1$ with probability p for all $i,j \in N$. This takes place independently of the other edges. In particular, random models of network formation include Bernoulli random graphs (Erdös and Renyi, 1959; Bollobás, 2001) and Markov graphs or p*-networks (Frank and Strauss, 1986; Wasserman and Pattison, 1996).

Small-world graphs
An algorithm for building links that differs from a pure Bernoulli random graph, has been suggested by Watts and Strogatz (1998) and Watts (1999). In particular, Watts and Strogatz (1998) wanted to generate a network that exhibits both relatively low diameter and nondegenerate clustering. Watts and Strogatz (1998) called this specific network structure they generated small-world networks. Small-world graphs have become very popular since the pathbreaking work of Watts (1999). In particular, small-world networks

combine two characteristic properties: a relatively high clustering and a relatively low average characteristic path length. In contrast to small-world networks, random networks exhibit a relatively low characteristic path length, but clustering is also low for these network types. Deffuant et al. (2002) investigated small-world networks (SWN) with respect to opinion formation and convergence.

Scale-free graphs

However, the way Watts (1999) generated small-world networks implies that the degree distribution of generated networks has a great deal more regularity and less variance than observed for real social networks.

To generate networks that exhibit degree distributions as observed in many social networks, one needs a process of link formation that differs from the pure Bernoulli type process, as observed distributions often exhibit fatter tails. The idea behind generating distributions with such fat tails dates back to Pareto (1896), for which the standard power law distribution is named. Accordingly, generated networks are called power law or scale-free networks. The characteristic property of scale-free networks is a high global centralization, where some nodes have an extremely high number of ties (these are called hubs). The distribution of vertices and the number of edges follows a power law: $P \sim z^{-\xi}$. A prominent example of a scale-free network is the Internet. Accordingly, if a computer virus reaches a hub it can spread and diffuse rapidly through the network. Analogously epidemiology and disease spreading via infection also take place rapidly within scale-free networks.

Hybrid networks

Finally, hybrid models to generate networks have been developed since purely random graph models do not exhibit the clustering or degree distribution that match many observed networks, while generated small-world networks do not exhibit observed degree distribution and power law or scale free networks do not exhibit observed clustering (Pennock et al., 2002; Kleinberg et al., 1999; Levene et al., 2002; Kumar et al., 2000; Cooper and A., 2003).

To construct information networks a modified α-model of Watts (1999) is applied, which can generate hybrid networks combining properties of scale-free, small-world, and random networks. The model is described in detail in Henning and Saggau (2009).

A specific property of a social network is clustering, this means the fact that the likelihood of a connection among two firms is correlated with the existing connections among firms. In detail, the higher the number of overlapping connections between a pair of firms, the higher the probability that these two will form a connection as well. How exactly clustering occurs is a very interesting topic in itself. In the Watt's α-model clustering is basically determined by a single parameter, α , which can vary between 0 and infinity. The lower the values for α the more a network is clustered.

Our modified α-algorithm includes the existence of stars, keeping the idea of clustering. In detail, we assume that only one star exists, and we vary the number of ties this star forms (for detailed network generation mechanism see in Henning and Saggau, 2009).

11.3.2. Agent-based Model of Knowledge Accumulation in an Information Network

Given the information network, Z, we assume the following simple three-stage procedure of knowledge accumulation in a network of firms, N:

1. **Generation and transmission of new knowledge**
   New knowledge is constantly generated in an external science sector or a leading region and randomly transmitted to a firm $i \in N$ of the network. In particular, we denote k_t as the new knowledge signal generated in period t and $K_T = \sum_{t=1}^{T} k_t$ the maximal accumulated knowledge available in the science sector. For simplicity, we assume that each knowledge signal equals 1: $k_t = 1 \quad \forall t$.
   A firm can absorb knowledge from the external source in each time period. Knowledge absorption is stochastic. In particular, we assume that in each time period, T, a firm can maximally learn one additional knowledge signal, where $p_i(m_i)$ denotes the probability that a firm i actually learns an additional signal. Formally, we assume that a firm will learn new knowledge if its effective invested resources, m_i^e, are above a threshold. Firms invest resources m_i to learn new knowledge from the external source. However, the transformation of invested (m_i) into effective resources (m_i^e) is a stochastic process. Technically, we assume that the threshold for each firm equals 1 and that effective resources are uniformly distributed over the interval $(0, m_i)$. Under this assumptions p_i results as: $p_i = (1/m_i)(m_i - 1)$.
   Further, let K_{iT} denote the stock of knowledge accumulated by firm i at time period T and let ΔK_{iT}^o denote the additional knowledge a firm absorbs form the external source then it holds:

$$K_{i1} = 0$$

$$\Delta K_{iT}^o = \begin{cases} 1, & \text{if } m_i^e \geq 1 \\ 0, & \text{if } m_i^e < 1 \end{cases} \tag{11.9}$$

2. **Diffusion of knowledge between regional firms**
   Firms communicate their accumulated knowledge to other firms within information network channels in each time period. Formally, let π denote the common conditional probability that firm i transmits its accumulated knowledge, K_{it}, to another firm j, if both firms have an established information tie, $z_{ij} = 1$. Then for each firm i we can define the set of received information in period T: $M_{iT} = \{K_{jT} + \Delta K^{o}_{jT} | d_{ijT} z_{ij} = 1\}$, where d_{ijT} is a random variable which is 1 with probability π and zero with probability $1-\pi$. To accumulate observed knowledge a firm has to process received knowledge. Again, the firms' processing capacity is limited. In particular, we assume that in each time period, T, a firm can maximally learn one additional knowledge signal. However, a firm is only able to learn from other firms if knowledge observed from another firm is new compared to the existing knowledge stock. Therefore, let ΔK^{N}_{iT} denote the additional knowledge a firm absorbs from knowledge spillovers then it holds:

$$\Delta K^{N}_{iT} = \begin{cases} 1, & \text{if } K_{iT} + \Delta K^{o}_{iT} \; < \; MaxM_{iT} \\ 0, & \text{if otherwise} \end{cases} \tag{11.10}$$

   where $MaxM_{iT}$ is the maximum of the set of received information, M_{iT}.

3. **Knowledge accumulation**
   Depending on the technological information received from knowledge spillovers within the network, as well as on the new knowledge absorbed outside the network, each firm accumulates its new knowledge, e.g., the new knowledge stock available in the next time period $T+1$, K_{iT+1}, results as:

$$K_{iT+1} = K_{iT} + \Delta K^{o}_{iT} + \Delta K^{N}_{iT} \tag{11.11}$$

11.3.3. An Intuitive Approach to our Model of Knowledge Accumulation

To get an intuitive understanding of the process of knowledge accumulation our simple model defines, consider stage 1 in our model. At stage 1 the knowledge stock of each firm in the network is 0 and the external source has generated the first knowledge signal k_1.

According to our defined process, the probability that a firm i will absorb this knowledge from the external source equals p_i. For simplicity we assume every firm has invested the same amount into its absorbtion capacity, e.g., for any firm i the probability to learn the new external information equals

p. Accordingly, the probability that the network collectively will learn the new signal results as:

$$1-(1-p)^n \tag{11.12}$$

Obviously, as long as p is small and the number of firms (n) is sufficiently large collective knowledge accumulation is much faster when compared to isolated accumulation of an individual firm. However, the long-term speed of collective learning crucially depends on the speed of knowledge diffusion through the network, that is on the number of firms that learn a new information signal via knowledge spillovers in a given period of time. For example, assume that the firms have no established network ties, i.e., $z_{ij}=0 \;\; \forall i,j \in N$. Then, obviously, the speed of knowledge accumulation just equals the speed observed for an isolated firm, that is p. On the contrary, considering a perfect connected network, i.e., $z_{ij}=1 \;\; \forall i,j \in N$ implies that the average speed of learning equals $1-(1-p)^n$. What will be the average speed of learning if we assume an imperfect communication network, i.e., $z_{ij}=1$ for some $i,j \in N$?

Consider a firm i has received the information signal in period $t=1$. Then this signal is communicated in the same period to all other firms j to which firm i has an established network tie. We define the subset of firms to which firm i has a direct network tie as his 1-path neighborhood, $N_i(1)$. Accordingly, we can define the 2-path neighborhood of a firm i as the subset of firms j to which firm i has indirect contact via one other intermediate. In general, we can define the r-path neighborhood as the following subset:

$$N_i(r)=\left\{ j \in N \middle| g(i,j) \leq r \right\} \tag{11.13}$$

where $g(i,j)$ is defined as the minimum path length between i and j (see above).

Now we define $A_{ir}=\left|N_i(r)\right|$ as the size of the r-path neighborhood of firm i. Further, let A_r denote the average size of the r-neighborhoods of all firms in the network.Then we can calculate the probability that the r-neighborhood of a firm i will receive the next new knowledge signal, P^r, as:

$$P^r=1-(1-p)^{A_r} \tag{11.14}$$

Obviously, the higher A_r for every communication round r the greater is the speed of knowledge diffusion.

The average size of r-neighborhoods depends on the average number of ties a firm has, i.e., on the average local network size, $\overline{z}$. At a first glance one could assume that the size of r-path-neighborhoods just equals $\overline{z}^r$, since every firm knows $\overline{z}$ other firms. However, it is easy to see that this approximation is too simple. First, our information network is symmetric. Therefore, every firm in the neighborhood of a 'parent' firm i has one tie with this parent firm. Therefore, the size of an *r*-neighborhood reduces to: $\overline{z}*(\overline{z}-1)^{r-1}$. Moreover, it is also clear that the total size of any *r*-neighborhood cannot be larger than the size of the total network. Furthermore, clustering has an impact on size of *r*-neighborhoods. This impact corresponds to the fact that the newly contacted firms and the other $\overline{z}-1$ firms contacted by its 'parent' firm form triadic configurations. Technically, the average cluster coefficient, γ, measures the share of triadic configurations occurring within a direct neighborhood.

Accordingly, it follows for the approximation of the average size of an *r*-neighborhood:

$$A_r = Min\left\{\overline{z}*\left[(\overline{z}-1)*(1-\gamma)\right]^{r-1}, n\right\} \tag{11.15}$$

Finally, the total network size also has an impact on the size of *r*-neighborhoods that are lower than the total network size. This follows from the fact that the smaller the total number of firms in a network the higher the probability that two firms in an *r*-neighborhood of a 'parent' firm form a tie to the same firm. However, it is tentative to find an analytical form correcting for this additional bias so that it is omitted here.[5]

Analogously, introducing stars into the network implies that the average size of *r*-neighborhoods, A_r, increases. For example, assuming that a central star that has a direct tie to every other firm implies that for every firm every *r*-path neighborhood with $r>1$ already includes the complete network. Accordingly, for networks with large stars knowledge diffusion will be much faster when compared to corresponding SWNs.

So far, we have only analyzed knowledge diffusion through the network. Next we combine diffusion with knowledge accumulation. To this end we first assume that knowledge accumulation can only take place in an *r*-neighborhood, i.e., only every *r*-time period firms have the opportunity to learn new knowledge from the external source, while knowledge spillovers occur every time period. Hence, firm maximal speed of knowledge accumulation corresponds to $1/r$. Moreover, firms learn in r-neighborhoods, thus the realized speed of knowledge accumulation results as:

$$\frac{1}{r}P^r \tag{11.16}$$

Accordingly, speed of knowledge accumulation in the *r*-neighborhoods crucially depends on knowledge diffusion in *r*-neighborhoods, that is the size of the *r*-neighborhood A_r. Finally, according to our model, firms do simultaneously learn in all *r*-neighborhoods. Thus, although we do not present a complete analytical solution approximating the speed of knowledge accumulation within our agent-based model, it is intuitively clear that overall knowledge accumulation will proceed faster is higher the speed of knowledge accumulation in all *r*-neighborhoods. Therefore, it follows directly from our expositions above that ceteris paribus the speed of knowledge accumulation increases the lower the clustering coefficient (γ) as well as the higher the global and the local network size, n and $\overline{z}$, respectively. Moreover, the speed should be significantly higher for star networks when compared to SWNs.

11.4. RESULTS

11.4.1. Simulated Information Networks

To be able to analyze the impact of network structures on speed of knowledge accumulation we have systematically simulated information accumulation in various SW- and hybrid networks. Networks have been generated using the modified α-model of Watts (1999).[6] A central parameter of this network generation algorithm is α which determines global network characteristics, i.e. clustering and characteristic path length (see Table 11A.1 in the appendix).

In particular, starting from a basic network comprising $n = 1000$ firms, who on average have $\overline{z} = 10$ direct contacts, we generated different SW-networks assuming α-values ranging from 0 to 10. Furthermore, we also generated different hybrid networks assuming one star with direct contacts in the range of 100, 250, 500, and 1000. Moreover, to analyze the impact of global network size, n, as well as the impact of the local network size, $\overline{z}$, we also ran simulations for $n = 3000$ and $n = 5000$, as well as for $\overline{z} = 15$ and $\overline{z} = 20$.

Overall, each specific network parameter constellation can be characterized by global and local network sizes ($n = 1000, 3000, 5000$; $\overline{z} = 10, 15, 20$), existences of a star (star = yes or star = no), the number of ties of the star (star-density = 100, 250, 500, 1000) and the α-parameter ($\alpha = 0, ..., 10$). Accordingly, we simulated knowledge accumulation in 180 different SW-networks and 720 different hybrid networks. Further, due to the random nature of our model we repeated each simulation run for each network parameter constellation 100 times, which means that all reported variable values generally correspond to the mean over 100 simulation runs.

Moreover, we set $\pi = 1$ for all simulation runs, thus we assumed that knowledge is transferred with certainty whenever firms have established an information tie, i.e. $z_{ij} = 1$. In essence, the main results regarding the impact of network structure on knowledge accumulation will not change if we assume different values for π, although the absolute speed of knowledge accumulation would significantly change.

Finally, the standard setting for p is 0.001 for all simulation runs. However, to analyze the impact of networks on speed of knowledge accumulation under different innovation intensities we also ran simulations assuming $p = 0.01$ and $p = 0.1$.

To get a better understanding of our main results, central network indicators have been calculated for all simulated networks. The mean values over the 100 simulation runs are reported in Table 11A.1 in the appendix.

11.4.2. Network Typology and Knowledge Accumulation

The key question of our simulation analysis is how different network typologies influence the accumulation of knowledge in a given network of firms. Accordingly, a relevant indicator is the average knowledge accumulated by an individual firm in the network.[7]

Let $\bar{K}_t$ denote the average accumulated knowledge in period t, while $w_t = \bar{K}_t / t$ denotes the average accumulated knowledge per time period.

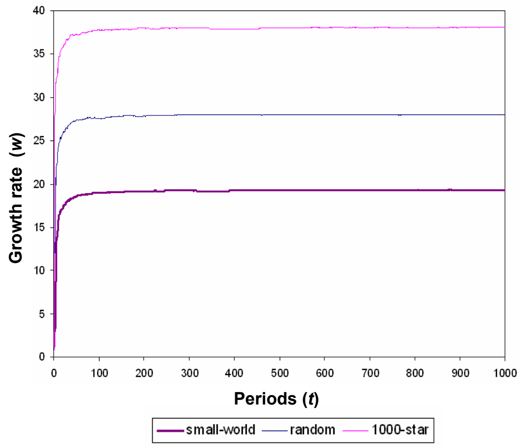
11.4.2.1. Impact of network types

Figure 11.1 presents average accumulated knowledge per time period for the different network types, namely a random network, a small-world network, and a 1000-star network (i.e. the star has 1000 ties), where the last two networks are both generated setting $\alpha = 7$.

As can be seen from Figure 11.1 for all network types knowledge accumulation follows a linear trend, where simulations normally approach stable linear growth rates after 100 time periods, while each simulation run includes 1000 time periods. Accordingly, we focus our further analyses on long-term linear growth rates, which we denote by w. Technically, in the following analyses reported values of w correspond to the linear growth rate calculated after 100 time periods.[8]

In particular, linear growth rates vary significantly over network types. In absolute terms the growth rate is the highest for the 1000-star network followed by the random network, while growth rate is the lowest for the SWN. Furthermore, as can bee seen from reported standard errors in Table 11.1, these differences in growth rates are statistically significant.

To see how knowledge accumulation is determined by global network structures beyond different network types, we next analyze how speed of knowledge accumulation varies for different α-values and different star sizes.



Source: Own calculations

Figure 11.1. Average growth rates $(w_t = \overline{K}_t / t)$ *of accumulated knowledge for different networks*

11.4.2.2. Clustering

By construction Watt's α-algorithm implies that the lower the α-value the more networks are locally clustered. As can be seen in Table 11.1 and as illustrated in Figure 11.2 clustering has a significant impact on knowledge accumulation.

In particular, within SW-networks growth rates increase by a factor of almost 8 from 3.4 to 27 percent for α-values ranging from 0 to 10, while for α-values above 10 simulated growth rates do not further increase significantly (see Table 11.1).

Please note that unconnected networks correspond to an extreme form of clustering. Accordingly, one can observe that growth rates steeply increase between $\alpha = 6$ and $\alpha = 7$, that is, between unconnected and connected networks. However, please note further that also local clustering generally decreases with α. As explained above, this fact is not perfectly reflected by corresponding global cluster coefficient (γ). Thus, for disconnected networks the effective network size is much lower than n depending on the size of the connected components. Accordingly, following our intuitive explanation above, observed low growth rates of knowledge stocks technically result from the low effective global network size in disconnected networks.

Table 11.1: Simulation results of knowledge accumulation assuming different network structures

		alpha										
		0	1	2	3	4	5	6	7	8	9	10
network type	growth											
SWN-1000	w	3.39	3.26	3.45	3.35	3.79	3.84	8.68	20.00	24.66	26.64	27.03
z=10	σ^2_w	5.62	4.24	5.34	3.72	5.24	7.24	18.19	0.14	0.11	0.10	0.10
	error	**0.07**	**0.07**	**0.07**	**0.06**	**0.07**	**0.09**	**0.13**	**0.01**	**0.01**	**0.01**	**0.01**
	w^m	8.35	7.73	8.52	7.72	8.52	9.22	12.95	20.77	25.31	27.24	27.59
SWN-1000	w	3.76	4.14	4.84	4.26	5.46	7.53	22.73	27.99	29.78	30.26	30.48
z=15	σ^2_w	5.05	6.13	6.33	4.68	8.23	12.55	0.14	0.09	0.09	0.08	0.07
	error	**0.07**	**0.08**	**0.08**	**0.07**	**0.09**	**0.11**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**
	w^m	8.62	9.01	9.62	8.71	10.52	12.59	23.45	28.62	30.42	30.75	31.07
SWN-1000	w	4.70	6.88	6.38	6.58	6.86	19.23	28.87	31.22	32.07	32.44	32.60
z=20	σ^2_w	5.78	13.20	12.06	7.70	7.93	9.67	0.10	0.07	0.09	0.09	0.08
	error	**0.08**	**0.11**	**0.11**	**0.09**	**0.09**	**0.10**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**
	w^m	9.55	12.87	12.29	11.18	11.68	20.75	29.52	31.86	32.61	32.97	33.14
SWN-3000	w	3.05	2.96	3.41	3.57	3.70	4.00	15.48	25.52	29.77	30.93	31.12
	σ^2_w	4.32	3.93	4.61	5.44	5.58	6.42	16.75	0.13	0.11	0.09	0.11
	error	**0.04**	**0.04**	**0.04**	**0.04**	**0.04**	**0.05**	**0.07**	**0.01**	**0.01**	**0.01**	**0.01**
	w^m	8.90	8.56	9.20	9.69	9.86	10.09	18.08	26.52	30.61	31.81	31.97
SWN-5000	w	2.79	3.22	3.35	3.53	3.47	4.72	20.37	27.59	31.09	31.79	31.96
	σ^2_w	3.66	4.54	4.55	4.84	4.37	9.43	1.42	0.12	0.12	0.12	0.12
	error	**0.03**	**0.03**	**0.03**	**0.03**	**0.03**	**0.04**	**0.02**	**0.00**	**0.00**	**0.00**	**0.00**
	w^m	8.80	9.75	9.46	9.95	9.59	12.46	21.84	28.66	32.07	32.71	32.85
100-star	w	11.08	14.47	15.18	13.39	12.38	10.98	20.58	23.48	25.53	28.41	29.63
	σ^2_w	94.19	94.96	95.58	88.89	89.87	85.64	38.07	0.15	0.11	0.11	0.09
	error	**0.31**	**0.31**	**0.31**	**0.30**	**0.30**	**0.29**	**0.20**	**0.01**	**0.01**	**0.01**	**0.01**
	w^m	21.82	22.98	23.14	22.10	22.04	21.90	23.45	24.23	26.18	29.08	30.21
500-star	w	33.03	33.68	34.97	34.54	34.87	35.05	35.48	35.48	36.55	37.02	37.45
	σ^2_w	11.42	0.11	0.07	17.72	15.95	0.10	0.09	0.11	0.07	0.05	0.03
	error	**0.11**	**0.01**	**0.01**	**0.13**	**0.13**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**
	w^m	33.90	34.13	35.49	35.58	35.81	35.50	35.92	35.93	36.95	37.40	37.92
1000-star	w	38.74	38.71	38.56	38.38	38.85	38.78	38.88	38.68	38.76	38.54	38.87
	σ^2_w	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.00
	error	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	w^m	39.09	39.07	39.01	38.71	39.22	39.15	39.32	39.05	39.18	39.00	39.23
1000-star	w	39.74	40.94	39.60	41.94	43.61	41.13	45.47	45.91	46.83	47.65	47.67
n=3000	σ^2_w	118.11	96.97	156.33	132.70	69.51	164.83	0.22	0.15	0.11	0.09	0.08
	error	**0.20**	**0.18**	**0.23**	**0.21**	**0.15**	**0.23**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**
	w^m	43.51	44.13	44.53	45.89	45.94	46.01	46.25	46.68	47.41	48.26	48.31
1000-star	w	33.46	35.47	34.59	35.64	36.08	40.31	45.29	46.16	48.22	48.33	48.41
n=5000	σ^2_w	319.43	292.27	340.93	342.98	344.17	230.05	6.24	0.21	0.15	0.13	0.12
	error	**0.25**	**0.24**	**0.26**	**0.26**	**0.26**	**0.21**	**0.04**	**0.01**	**0.01**	**0.01**	**0.00**
	w^m	44.45	45.03	45.98	46.82	47.22	47.25	46.48	47.09	48.95	49.02	49.16
3000-star	w	48.97	49.18	49.05	49.28	49.02	49.11	49.06	49.13	49.16	48.97	49.07
	σ^2_w	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	error	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	w^m	49.52	49.65	49.54	49.73	49.46	49.58	49.62	49.66	49.61	49.51	49.63
5000-star	w	50.50	50.49	50.50	50.45	50.47	50.29	50.39	50.50	50.42	50.42	50.53
	σ^2_w	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	error	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
	w^m	51.05	50.97	50.97	50.92	50.93	50.88	50.91	50.92	50.95	50.93	51.04

w = average knowledge accumulation | σ^2_w = variance of average knowledge accumulation

w^m = highest information | error = standard error

Source: Own calculations

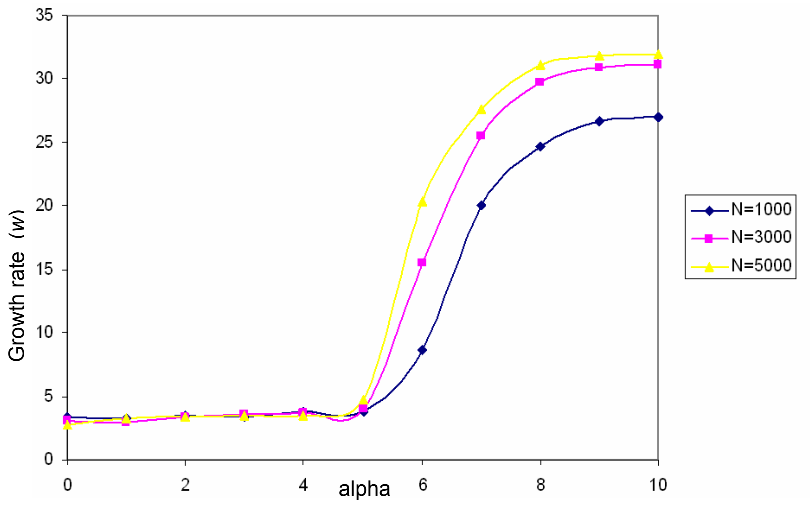


Figure 11.2. Clustering and growth rates (w in percent) of accumulated knowledge in SWNs

Clustering also has a significant impact on speed of knowledge accumulation within connected networks as growth rates increase by 35 percent from 20 to 27 percent, ranging α from 7 to 10. Note that simulated differences between different clustering structures are statistically significant given the extremely low standard errors for average growth rates reported in Table 11.1.

The impact of clustering is clearly attenuated in hybrid networks including a star (see Figure 11.3).

For example, for a 100-star network, i.e., a star with 100 ties, increasing clustering from (α=10) to (α=7) implies only a decrease of the growth rate (w) by a factor of 1.26 from 23.5 to 29.5 percent. Moreover, the larger the star the more the impact of clustering vanishes, e.g., for a central star connected to every other firm (star-1000), clustering has almost no impact on knowledge accumulation (see figure 3). Further, star networks are generally completely connected (see Table 11.1). Only the star-100 and -250 networks are not fully connected for α-values below 7, and here we also observe that knowledge accumulation is significantly slower for unconnected networks.

Finally, comparing Figures 11.2 and 11.3, one can clearly see that although clustering generally reduces the speed of knowledge accumulation in both network types, in quantitative terms the impact is much higher for SWNs.

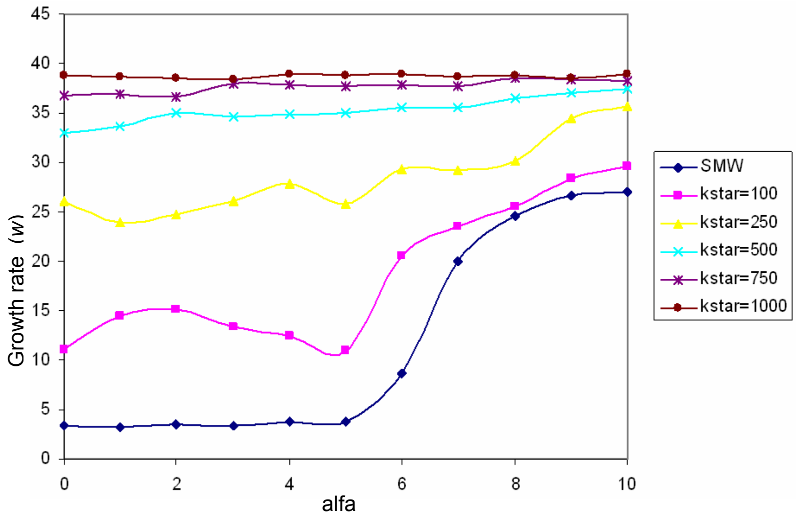


Figure 11.3. Clustering and growth rates (w) of accumulated knowledge in star networks

Again, all simulated differences between different clustering structures are statistically significant given the extremely low standard errors for average growth rates reported in Table 11.1.

11.4.2.3. Size of the star

Further, Figure 11.3 clearly shows that the larger the size of the star the higher the speed of knowledge accumulation. This clearly results from our intuitive interpretation of our agent-based model, since the higher the number of ties of the star the larger are c.p. the *r*-neighborhoods and thus the faster knowledge diffuses through the network accelerating knowledge accumulation. Obviously, the existence of a central star being connected to every node in the network (1000-star in figure 3) implies that the 2-path neighborhood already includes the complete network. In this case the growth rate of knowledge accumulation approximates 0.5 as the network learns a new signal approximatively every second time period. An important question is how central stars are conceivable in real firm networks. One idea might be that large firms which are technological leaders become central stars in firm information networks. However, alternative candidates for central stars could be consulting companies or business associations. Empirically, it appears more conceivable for the latter to maintain many network ties than a large firm (Henning et al., 2008).

11.4.2.4. Global network size

As can bee seen in Figure 11.2, the global network size for the SW-network significantly increases the speed of knowledge. However, there is a significant negative interaction effect among clustering and network size, where impact of increasing network size almost vanishes when clustering is sufficiently high. For example, for $\alpha = 10$, indicating a cluster coefficient of $\gamma = 0.115$, an increase of global network size from 1000 to 5000 implies an increase of the speed of knowledge accumulation from 0.27 to 0.32, while for $\alpha = 0$ almost no increase can be observed.

A similar increase of knowledge growth induced by a higher global network size can be observed for star networks (see Table 11.1). Note that only n is increased, while the local network size, i.e., the average number of network ties, $\bar{z}$, is held constant, which means the global network density decreases. Moreover, the global cluster coefficient, γ, decreases, while the characteristic path length L increases with global network size (see Table 11A.1 in the appendix). Accordingly, the impact of increased global network size on knowledge accumulation can be separated into two different effects. A direct effect results, holding clustering constant. The explanation for the direct effect of global network size results from the fact that a larger network size implies c.p. that the size of all k-path neighborhoods increases, since the probability of redundant network ties decreases with n.

Beyond the direct effect, shifting global network size also has an indirect effect, since an increased global network size implies less clustering for any given α-value (see Table 11A.1 in the appendix). This indirect effect on knowledge accumulation via reduced clustering is also positive.

A similar effect of network size can be observed for star-networks (see Table 11.1), although we do not present this effect graphically to save space. However, especially for large stars, the effect of network size on knowledge accumulation is attenuated (see Table 11.1).

11.4.2.5. Local network size

Finally, obviously the diffusion of knowledge through the network is the faster the higher the average number of direct ties, that is the larger average local network size, $\bar{z}$. As Table 11.1 shows, local network size has a significant impact on average knowledge growth rates. Again, a strong negative interaction effect with clustering can be observed, where the positive impact of a higher average local network size increases with α. Hence, technically, the optimal speed of knowledge would result for a perfectly connected network, where every firm is connected to every other firm. In contrast, economically the cost of establishing network ties must be taken into consideration, implying that completely connected networks are not necessarily most efficient.

So far, we have only analyzed how an increase of the average local network size affects average knowledge accumulation in the network. Another interesting question is how individual local network size, z_i, affects the speed of knowledge accumulation of individual firms. Here, the exogenous assumption of Goyal and Joshi (2003) would clearly imply that firms with a higher local network size should also observe a higher speed of knowledge accumulation. However, our simulation results clearly contradict this implication. As table 1 shows the difference between the maximal accumulated knowledge in the network and the average accumulated knowledge is lower or equal to 1 for all connected networks. Moreover, in the long run this difference is constant over time, which implies that all firms in a connected network accumulate knowledge at the same speed.

Interestingly, the same results can be observed for star-networks (see average and maximal knowledge stocks in Table 11.1). Even the star with a much larger local network size cannot accumulate faster knowledge when compared to the average firm in the network. Moreover, note that the variance of total information accumulated after 100 time periods across all firms in the network as reported in Table 11.1 (σ_w^2) is extremely low. Accordingly, this variance indicates that firms in a connected network hardly differ in their accumulated knowledge stocks, i.e., the firms' accumulated knowledge differs less than 1 unit from the average accumulated knowledge. It is only in disconnected networks that firms differ in their accumulated knowledge stocks as well as speed of knowledge accumulation in the long run (see Table 11.1). Further, reported standard errors of the variance of accumulated knowledge in the network are extremely low, indicating that a low variance of knowledge stocks across firms and a constant growth rate of knowledge within connected networks is a stable finding of our simulation analyses. Finally, we also calculated correlation coefficients between firm's local network size (z_i) and firm's accumulated knowledge stock in each time period $t = 1, .., 100$ [9]. Calculated correlation coefficients are extremely low for all SWN as well as for star networks. For example, for our baseline SWN ($n = 1000$, $\alpha = 7$) calculated correlation coefficients range between 0.018 and 0.09, while for a 1000-star network calculated correlation coefficients range from 0.29 to 0.35. Thus, to give a more intuitive interpretation of these correlation coefficients, we have also calculated the estimated parameter of local network size resulting form a simple linear regression of accumulated knowledge on local network size. This parameter corresponds to the relation of the covariance of local network size with accumulated knowledge divided by the variance of local network size. Thus, our results imply a regression parameter around 0.001 for the star networks and around 0.025 for the SWN indicating a neglectable marginal effect of local network size on accumulated knowledge.

These results clearly contradict the assumption of Goyal and Joshi (2003) and seem surprising at first glance. However, given our assumed mechanism of knowledge accumulation it follows quite plainly that knowledge accumulation as defined by our model is basically a collective process. The probability that an individual firm adopts new knowledge from the external source is extremely low $(p = 0.001)$; thus, speed of knowledge accumulation can be accelerated only if the network collectively learns.

Even a central star can only accumulate knowledge at a higher rate than 0.001 if his neighborhood accumulates at the same rate as the star. Moreover, note that in general the speed of knowledge accumulation is the same in the long run for all firms in a connected network. Moreover, assuming all firms have no knowledge (or the same amount of knowledge) at the beginning, the maximal difference of accumulated knowledge between firms is always limited to the maximal path length in the network. For the networks considered in our simulation analyses this differences hardly exceeds 1 as can be seen from table 1. How can we intuitively understand this result? Basically, two different processes can be distinguished in a connected network. On the one hand the probability that the complete network learns a new signal. Obviously, this probability crucially depends on the number of firms that hold the maximal knowledge stock in the network. On the other hand the number of firms that hold the maximal knowledge in the network in each time period crucially depends on the speed of diffusion of knowledge through the network. In essence the diffusion process can be described by a recursive formula (see Rapoport, 1953), which approximatively corresponds to eq. (15) above. However, as long as the second process is faster than the first, it follows that every new knowledge learned by the network completely diffused through the network before the network learns the next new signal. For example, for networks between 1000 and 5000 with a local network size of 10 and clustering coefficient lower than 0.45 a signal completely diffuses through the network within 3 time periods. Accordingly, as long as the maximal speed of knowledge accumulation is not above 1/3 (which basically implies that a firm learns every 3 time periods a new signal), the expected maximal difference of knowledge stocks between firms would be 1. For example, take a 1000-star network, where every firm is connected to the firm in the center of the star. In this case any new knowledge signal received by any firm except the star diffused through the complete network within two time periods, while the maximal growth rate is 31 percent (see Table 11.1). Therefore also for the 1000-star the maximal difference of the accumulated knowledge between any pair of firms is 1.

Of course, we must admit that our results are specific inasmuch as we are assuming a specific process of knowledge accumulation. However, we argue that the assumed process, especially the feature that individual learning is much harder than imitating other firms, corresponds sufficiently to many processes of knowledge accumulation that can be found in reality.

11.4.3. The Impact of Networks Under Different Innovation Intensities

So far we have analyzed to what extend network structures have an impact on knowledge spillovers and thus on knowledge accumulation, assuming a specific innovation intensity, i.e., the probability that an individual firm acquires new knowledge in a given time period was set to $p = 0.001$. However, the impact of network structures obviously depends on the innovation intensity, that is, the firms' investment into absorption capacity, m_i. For example, it is obvious that if the innovation intensity is extremely low, e.g., p approximates 0, individual firms quasi never learn. Accordingly, knowledge spillovers and thus network structures become irrelevant. On the other hand, assuming an extremely high innovation intensity, knowledge spillovers and thus network structures become less important, as in essence learning becomes a independent individual process.

In Figure 11.4 we report simulated knowledge growth rates assuming different innovation intensities, e.g., ranging p from 0.001 to 0.01 and 0.1. As can be clearly seen for $p > 0.1$, which means ($log \quad p > -1$), innovation intensity is sufficiently high that clustering has little impact on speed of knowledge accumulation. Analogously, the impact of a central star is significantly reduced as speed of knowledge is only slightly higher for the 1000-star-network when compared to SW-networks (see Figure 11.4).

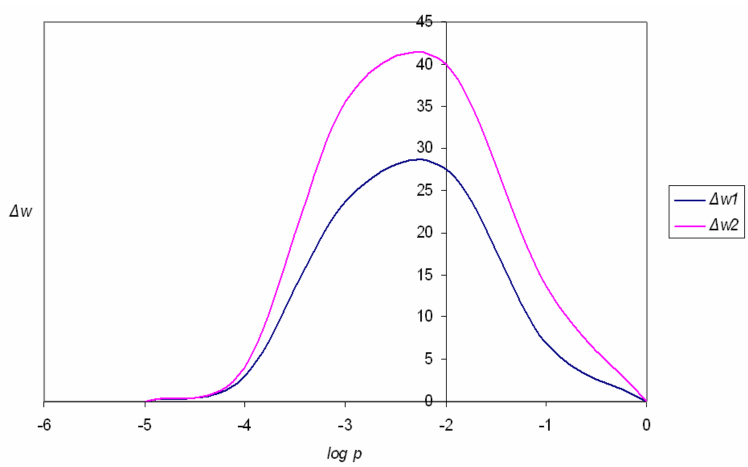


Figure 11.4: Impact of network structures under different innovations intensities

The lower line $(\Delta w1 = SWN_{10} - SWN_0)$ displays the difference between two SW-networks where the first is a SW-network created with $\alpha = 10$ while the second SW-network was constructed with $\alpha = 0$. The second curve $(\Delta w2 = 1000 - star_{10} - SWN_0)$ shows the difference for 1000-star-network with $\alpha = 10$ and a SW-network with $\alpha = 0$.

11.5. CONCLUSION AND OUTLOOK ON FUTURE RESEARCH

Stimulated by new evolutionary economic approaches to innovation and technical progress, this paper aims to contribute to a better understanding of technological progress via explicitly modeling the process of knowledge diffusion in a given network of firms.

Although, the importance of established informal and formal interfirm network structures for knowledge accumulation has clearly been recognized in the new ACE-literature on innovation, it is still fair to conclude that only very few studies exist analyzing the role of networks in knowledge spillovers and knowledge accumulation. In this regard the paper certainly contributes to the existing literature.

In particular, the paper contributes to theoretical studies in the innovative field of networks and economics that explore the emergence of stable and efficient R&D networks in a game-theoretical framework. In particular, the fundamental assumption that the number of direct network ties has a positive impact on firms' profit, made by Goyal and Joshi (2003), is challenged by our results. Results of our detailed simulations clearly imply that although at the macro level the average local network size significantly increases speed of knowledge accumulation in the complete firm network, at the micro level individual firms with a larger number of ties do not accumulate knowledge at a higher speed when compared to firms with an average number of network ties. Even a central star does not accumulate knowledge at a higher speed compared to the average firm in the network as long as networks are connected. Only if a network is disconnected do firms differ in their speed of knowledge accumulation. However, the absolute speed is extremely low for unconnected networks when compared to connected networks. Thus, individual firms have no incentives to maintain a large number of direct information ties. On the contrary, a free-rider problem might even arise regarding the establishment and maintenance of network ties. These results have major implications for the analyses of stable and efficient network structures. In particular, major results of by Goyal and Joshi (2003), might change in the framework of our model. We consider these implications as an interesting starting point for future research in the field of economics of networks and knowledge spillovers. Thus, future research might focus on how or under which condition this kind of free-rider problem occurs and influences the emergence of network structures.

Furthermore, given the fact that the majority of studies in new growth theory ignores the process of knowledge diffusion, our theory has also potential implications for regional economic growth theory. For example, models of the new economic growth theory imply that catching up generally occurs, since speed of knowledge transfers between leading and follower regions are monotonically increasing in the technological gap between a leading and a follower region (Barro,1995; Grossman and Helpman, 1991). However, empirical studies hardly confirm the neoclassical hypothesis on catching up (Rogers, 2003). In response to the lack of empirical support of the neoclassical catching up hypothesis, some scholars suggest a non-monotonic relation between knowledge spillovers and technological gap (Verspagen, 1991; Rogers, 2003). Basically, the idea behind the assumption of a non-monotonic relation is that if the technological gap is too high, implying that the follower region is unable to decode the knowledge of the leading regions, little learning occurs. However, all of the existing approaches on catching up focus on the individual firm level. In this regard our theory extends existing theories of catching up as it understands knowledge accumulation basically as a collective process. Accordingly, beyond the absorption capacity of individual firms, speed of knowledge accumulation and hence catching up are crucially determined by interfirm network relations within the follower region. Note that in the framework of our model, assuming a low innovation intensity at individual firm level, catching up only occurs if information networks are extremely dense or extremely centralized or are characterized by an extremely large average local network size. Accordingly, our model also has interesting implications for regional development policy. In particular, beyond policies aiming to improve the firms' capacity to adopt knowledge from the leading region, such as improving trade openness and the establishing international business links or sponsoring international academic exchange and studies abroad , it highlights the importance of policies improving establishment of interfirm network ties within the follower region. Examples for the latter are promotion of establishment of business associations or business meetings (exhibitions), provision of library services, technical journals, etc,.

Moreover, our study provides a theoretical foundation of existing empirical studies using only vague measures of structural (network) relations among firms or other indicators of social infrastructure as proxies for social capital (Knack and Keefer, 1997) as it generates testable hypotheses regarding the relation of quantitative network indicators of business networks with the average growth rate of knowledge and thus c.p. with average technological progress observed for a finite set of firms.

However, to focus on the role of information networks in knowledge spillovers, the paper applies a rather simple model neglecting other important factors of technological progress, e.g., innovation processes resulting from the firms' in-house R&D. We claim that networks also play a role regarding innovation processes; however, this has not been analyzed in this paper and thus, we consider this as one interesting topic for future research. In particular, it would be interesting to analyze how knowledge accumulation speeds up assuming a combined heterogeneity of local network size and investment in absorption capacity, that is a network including central stars specialized in absorption of external knowledge.

Finally, our approach simply assumes that accumulated knowledge is the key factor of technical progress, although beyond knowledge technical progress often requires investment in new capital goods. This investment is often characterized by a fundamental uncertainty, i.e., firms must form beliefs regarding future states of the world without knowing ex ante the set of all possible future contingencies. Therefore, the firms' decision to invest in innovation can be better captured by agent-based approaches assuming strong substantive and procedural uncertainty than by dynamic optimization models with Bayesian updating or even perfect foresight. Social networks also have a significant impact on agents' belief formation and thus on firms' investment decision regarding innovative projects. These aspects of the impact of social networks on innovation have not been considered in this paper; however, this as an other interesting topic for future research.

11A. APPENDIX

11A.1. Indicator Table

Table 11A.1. Network indicators

alpha		0	1	2	3	4	5	6	7	8	9	10
network type	**indicators**											
SMW-1000	γ	0.62	0.72	0.76	0.80	0.82	0.83	0.83	0.74	0.44	0.20	0.12
	L	1.74	2.00	2.03	2.30	2.04	2.57	3.15	5.77	3.81	3.43	3.35
	σ	2.31	2.35	2.74	2.80	2.77	2.79	2.93	2.63	3.03	2.90	2.67
	lc	6.90%	5.89%	8.40%	7.50%	5.40%	11.50%	18.09%	100.00%	100.00%	100.00%	100.00%
SMW-3000	γ	0.63	0.71	0.77	0.80	0.82	0.83	0.80	0.61	0.26	0.08	0.03
	L	1.76	1.91	2.06	2.13	2.23	2.43	11.71	5.19	4.08	3.80	3.76
	σ	2.24	2.48	2.66	2.71	2.78	2.79	2.78	2.90	2.87	2.50	2.32
	lc	2.62%	2.82%	2.87%	2.95%	3.14%	4.73%	94.76%	100.00%	100.00%	100.00%	100.00%
SMW-5000	γ	0.63	0.71	0.77	0.80	0.82	0.82	0.78	0.53	0.18	0.04	0.01
	L	1.73	1.92	2.06	2.14	2.22	2.67	9.66	5.12	4.21	4.01	3.97
	σ	2.23	2.47	2.63	2.73	2.77	2.80	2.79	2.92	2.76	2.38	2.28
	lc	1.68%	1.81%	1.88%	2.11%	2.09%	4.86%	99.25%	100.00%	100.00%	100.00%	100.00%
100-star	γ	0.56	0.64	0.69	0.72	0.76	0.78	0.80	0.72	0.46	0.22	0.14
	L	2.44	3.11	2.78	2.79	2.87	3.19	4.68	4.98	3.75	3.33	3.26
	σ	5.49	5.92	6.37	6.93	7.18	6.68	6.79	6.22	6.63	8.89	9.06
	lc	45.80%	51.80%	38.90%	48.50%	45.40%	52.70%	67.20%	100.00%	100.00%	100.00%	100.00%
500-star	γ	0.39	0.41	0.45	0.48	0.53	0.58	0.62	0.64	0.55	0.49	0.48
	L	2.40	2.40	2.30	2.30	2.24	2.26	2.23	2.26	2.14	2.07	2.06
	σ	19.89	20.49	22.07	23.15	24.48	23.95	24.57	24.08	26.91	29.00	29.32
	lc	100.00%	98.70%	95.00%	100.00%	100.00%	98.20%	100.00%	100.00%	100.00%	100.00%	100.00%
1000-star	γ	0.34	0.34	0.35	0.36	0.38	0.42	0.47	0.54	0.63	0.71	0.72
	L	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
	σ	31.43	31.43	31.43	31.44	31.44	31.44	31.45	31.45	31.45	31.45	31.45
	lc	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
3000-star	γ	0.33	0.34	0.34	0.34	0.35	0.37	0.39	0.44	0.50	0.58	0.59
	L	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
	σ	54.66	54.66	54.66	54.66	54.66	54.66	54.67	54.67	54.67	54.67	54.67
	lc	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
5000-star	γ	0.33	0.33	0.34	0.34	0.34	0.36	0.37	0.40	0.45	0.52	0.53
	L	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
	σ	70.63	70.63	70.63	70.63	70.63	70.63	70.63	70.63	70.63	70.63	70.63
	lc	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

γ = clustering coefficient *L* = characteristic path length σ = centralization *lc* = largest component

Source: Own calculations

11A.2. A Construction Algorithm for Information Networks

To construct an information network, the modified α-model (Watts, 1999) is applied which is able to generate hybrid network combining properties of scale-free, small-world and random networks.

According to this procedure, first a basic measure of the propensity that two vertices connect, μ_{ij}, is assumed. Given the basic probabilities of inter-firm relationships, μ_{ij}, a specific property of a social network relation is clustering, i.e., the fact that the likelihood of a connection among two firms is correlated with the existing connections among firms. In detail, the higher the number of overlapping connections between a pair of firms, the higher the probability that these two will form a connection as well.

How exactly clustering occurs is a very interesting topic in itself. However, we leave this interesting question for future research and simply assume that clustering occurs and can be implicitly incorporated into our network construction via defining the following transformation of the propensity measure, μ_{ij}:

$$R_{ij} = \begin{cases} 1, & b_{ij} \geq z_i \\ \left[\dfrac{b_{ij}}{z_i}\right]^{\alpha} (1-\mu_{ij}) + \mu_{ij}, & z_i > b_{ij} > 0 \\ \mu_{ij}, & b_{ij} = 0 \end{cases} \tag{11A.1}$$

In equation (11A.1), b_{ij} denotes the number of overlapping ties between firms i and j. According to the α-model of (Watts, 1999), the parameter α determines the clustering of the network. For the special case of $\alpha = 0$, a highly clustered network results. Note that, for $\alpha = 0$, R_{ij} becomes 1 whenever two vertices have at least one common connection, $b_{ij} > 0$.

On the other hand, the larger α the more R_{ij} converges against the basic propensity, μ_{ij}. Thus, as long as we assume that μ_{ij} is sufficiently small for all pairs, a random graph results.

Given the definition of the propensities R_{ij}, the following construction algorithm can be defined:

1. Compute probabilities, μ_{ij}, for all pairs $(i, j) \in N^2$.
2. Randomly select a firm i. .
3. Compute R_{ij} for all other vertices $j \in N$, where $R_{ij} = 0$, if i and j have already a connection.
4. Sum R_{ij} over all j and normalize each to obtain $Q_{ij} = R_{ij} / \sum_{k \neq i} R_{ik}$. Q_{ij} can be interpreted as the probability that i will connect with j. Thus, two mechanisms can be applied to select a connection, j, randomly with probability Q_{ij}. (a) Divide the unit interval $(0,1)$ into $(n-1)$ half-open subintervals with length Q_{ij}. Generate a uniform random variable on $(0,1)$. It must fall into one of the intervals, Q_{ij}. Connect i with j. the interval that the random variable falls into. (b) randomly select a vertex j, with probability $1/(n-1)$, randomly select a uniform variable on $(0,1)$, if it is lower or equal than Q_{ij} connect i with j. Otherwise start the process again.

This procedure (1) to (4) is repeated until the predetermined number of edges $M = \sum_i z_i$ has been constructed. The vertices i are chosen randomly, but once a vertex i has been chosen it may not be chosen again until all other vertices have taken their turn.

Barabási and Albert (1999) provided a model which constructs scale-free networks; starting with a small number of vertices b_0 and adding a new vertex at every step. The new vertex will then be connected to b other vertices, where the probability in connecting to a vertex depends on the number of already existing edges of this vertex. This principle is called preferential attachment, which has similarities to b_{ij} in the α-model of Watts (1999).

The α-model presented in this paper is used to generate small-world networks.

An extension of the α-model allows us to analyze scale-free networks, including a number of s 'stars', which are agents that hold a higher degree than the average agent.

The construction algorithm for the network with stars is a nested combination of the original α-model of of Watts (1999, p. 47) where the density of the network is maintained to allow for a comparison of the network types:

1. Random selection of s stars characterized by z_s information ties.
2. Recalculation of the degree for non-stars and construction of the network by using the α-model algorithm until $M - s\,z_s$ edges are distributed.
3. Application of the α-model algorithm, selecting only stars to distribute the remaining network ties, i.e., only the stars are allowed to choose ties.

This algorithm keeps the idea of clustering according to existing ties, which is similar to Watts' approach, which seems to be logical for the stars as well. In detail, we assumed that only one star exists, where we vary the number of ties this star forms.

NOTES

1. Although their empirical test did not provide significant evidence for an impact of their selected social capital indicators on economic growth (Knack and Keefer, 1997).
2. Alternatively, the process of absorbing external knowledge could also be interpreted as original generation of new knowledge, e.g., as a creative process via learning-by-doing or via a firms' own research and development activities.
3. However, the model of knowledge accumulation derived here also applies if we assume new knowledge is generated via learning-by-doing instead of assuming new knowledge is exogenously generated in a science sector or leading region.

4. A main interest in this regard is how the firms' ability to absorb knowledge spillovers affects the firms' investment into their ability to absorb external knowledge outside of the network. Obviously, for the latter a free rider problem arises in the presence of knowledge spillovers. However, we postpone this interesting topic for later research and do not investigate the free-rider problem inherent in the firms' investment into absorption capacity in this paper.
5. However, in another paper we apply the seminal work of Solomonoff and Rapoport (1951) and Rapoport (1953), who were one of the first to analyze how diffusion of information is determined by global network structure, to derive a more accurate approximation of the size of r-neighborhoods.
6. The detailed algorithm is described in Appendix 11A.2.
7. Of course, as one anonymous referee pointed out, other indicators, e.g., the maximal accumulated knowledge in the network, are also conceivable. However, we argue that average accumulated knowledge is an appropriate indicator as far as the impact of knowledge accumulation on regional economic growth is concerned. As reported below, in our specific case maximal accumulated knowledge does not differ significantly from average knowledge, thus both indicators have the same economic implications. We thank one anonymous referee for this comment.
8. Please note that for all undertaken simulation analyses growth rates did not significantly change after 100 time periods as can be seen in Figure 11.1. Therefore, calculated long-term growth rates (w) presented in Table 11.1 as well as in Figures 11.2–11.4 generally correspond to growth rates calculated in time period 100. Moreover, as we undertook for every network parameter constellation 100 simulation runs, long-term growth rates (w) presented in Table 11.1 as well as in Figures 11.2–11.4 are the average long-term growth rates calculated over this 100 simulations. Analogously, we calculated the variance of the long term growth rate across firms within the network, where in Table 11.1 the reported variances σ_w^2 correspond again to the average variance calculated over the 100 simulations undertaken for each network parameter constellation. Finally, the standard error of the average long term growth rate (error) presented in Table 11.1 has been calculated based on the 100 repeated simulation runs for each network constellation.
9. We thank an anonymous reviewer for suggesting the additional calculation of correlation coefficients to underline our main finding

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