10. Dualism and the big push

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10.1. INTRODUCTION

The concept of the poverty trap has been used fruitfully since the very dawn of development economics, and can implicitly be traced back even to Adam Smith's 'Early draft of part of the Wealth of Nations'.[1] Beginning with the seminal paper of Rosenstein Rodan (1943), the idea that underdevelopment could constitute a state of equilibrium thrived with Nurkse's *vicious circle of poverty* (1953) and Nelson's *low-level equilibrium trap* (1956). Several mechanisms, essentially concerning increasing returns coupled with pecuniary externalities or demographic traps, were from time to time held responsible for creating a multiplicity of equilibria, and possibly preventing the spontaneous development of certain economies.[2]

Despite the deep interest enjoyed by the so-called 'high development theory' in the 1950s, its predominantly discursive argumentation together with the difficulty reconciling increasing returns with competitive market structures[3] contributed to its decline in favour of the more analytically rigorous paradigm described by Solow (1956) and Swan (1956). Notwithstanding many important contributions on the role of increasing returns and learning by doing, during the 1960s the mainstream approach to growth became that of the neoclassical convex economy converging to a stable and unique steady state. Additionally, attention shifted from the 'developmental perspective' – emphasizing the role of 'sectoral balances', as well as the dualistic nature of developing countries' economies – to an aggregate growth perspective – focusing more on reproducible factors accumulation, and on the determinants of the steady state. It is worth noting here that the choice of an aggregate model rejects by definition the role of relative price changes, and overlooks the empirically-founded recognition that economic growth goes hand in hand with structural change.[4] Clearly, this latter flaw can be *a fortiori* misleading when analysing those economies that are indeed undergoing a process of industrialization and not of 'homothetic growth'.[5,6]

Regardless of the possible limits of aggregate models, the mainstream approach has played a key role in bringing back to the forefront of attention the issue of increasing returns, along with their crucial implications for multiple equilibria. The twist away from the traditional paradigm of the convex economy occurred in the mid 1980s, when endogenous growth theory stressed the role of knowledge and human capital. Assuming increasing returns to reproducible factors, where the latter typically include knowledge, responded to the need to rationalize two elements of industrial economies that were basically traced back to exogenous elements in Solow's conceptual framework: the persistence of growth even after the capital–labour ratio has reached fairly high levels, and the continuity (or possibly even the acceleration) of technical change. Endogenous growth theory was mostly concerned with issues other than explaining the take-off of initially poor countries. Consequently, it focused more on the properties of the steady state path, rather than on the possible obstacles to industrialization. This different *raison d' être* also explains why early endogenous growth models featured predominantly a one-sector or 'quasi-one-sector' set-up,[7] omitting by definition any possible role for labour reallocation and structural change.

In any case, in the mid 1990s concepts like poverty traps, structural change and multiplicity of equilibria were recovering a central role in the debate on economic growth, leading to what has been called a 'counter-counterrevolution in development theory' (see Krugman, 1992). On the one hand, it was shown that even in the standard neoclassical set-up (one sector with convex technologies operating under perfect competition), multiple equilibria cannot be excluded *a priori* once empirically significant elements such as heterogeneity in saving behaviour, low elasticity of technical substitution, or capital market imperfections are taken into account (see Galor, 1996; and later Azariadis, 2006; Easterly, 2006; Kraay and Raddatz, 2007). On the other hand, advances in the theoretical analysis of non-perfectly competitive market structure, jointly with a new stream of literature on structural change, caused a revival of categories inherited from the 'high development theory'.[8]

The renewed interest in poverty traps also came under pressure from the empirical literature, which increasingly questioned the validity of the neoclassical paradigm of conditional β-convergence, in favour of more complex dynamics able to generate convergence clubs and twin-peaked distributions. Cross-country regressions have long confirmed that economies tend to converge to their own steady state at a rate consistent with the 'augmented Solow model', once controlling for the determinants of the steady state itself: typically the saving rate, the initial level of human capital, political stability and degree of price distortion (see, among others, Barro, 1991; Mankiw et al., 1992; Barro and Sala-i-Martin, 1995; Sala-i-Martin, 1996; Easterly, 2006). Despite this, several econometric works accounting for parameter heterogeneity across countries (rather than relying on a common linear specification, as in standard growth regressions) found evidence of *convergence club* formation (see Durlauf and Johnson, 1995; Durlauf et al., 2001 and 2005). On the other hand, the existence of convergence clubs also seems confirmed by several studies based on non-parametric inference about the cross-country distribution of GDP per capita, and on the 'distribution dynamics' of Markovian growth processes (see Bianchi, 1997; Quah, 1993 and 1996; Ros, 2000; Azariadis and Stachurski, 2005; Azariadis, 2006). While not necessarily incompatible with neoclassical growth models, the existence of convergence clubs seems to be at odds with the traditional paradigm of the convex economy *à la* Solow, while it immediately rationalizes the observed absolute σ-divergence across countries.

In light of the long-standing debate summarized above, in this chapter we aim at building a theoretical model able to reconcile the neoclassical theory of growth (highlighting the role of reproducible factors accumulation) and the 'developmental perspective' (with its emphasis on structural change entailed by industrialization). In particular, we retain from the early development literature the dualistic set-up with its asymmetric treatment of agriculture and industry, in order to highlight the role played by factors reallocation in the early phases of industrialization.[9] We do so by developing a specific-factor macro model *à la* Ricardo–Viner–Jones, which may display multiple equilibria and a poverty trap under plausible parametrization. In several respects our set-up resembles the 'Rosenstein-Rodan–Leibenstein model' formulated in Ros (2000); however, we depart from it in adopting a sociological theory of efficiency wage and eliminating recourse to the Lewisian labour surplus. These choices allow us to generalize Ros's results while adopting a fully neoclassical formalization – with flexible prices, perfect competition and under the marginal theory of distribution.

The chapter is organized as follows: Section 10.2 outlines the macro model and the determination of the equilibria, Section 10.3 explains the effect of exogenous technical progress (here intended as a parametric increase in sectoral TFP) in each of the two sectors, and Section 10.4 concludes.

10.2. THE MODEL

10.2.1. Preferences

The economy consists of two sectors, agriculture and industry, producing respectively food – a consumption good – and manufactures, which can be alternatively consumed or invested. A traditional Cobb–Douglas utility function is used to describe consumers' preferences across goods:

$$U=\left(X_a^c\right)^{\alpha}\left(X_i^c\right)^{1-\alpha}$$

where X_a^c and X_i^c are respectively the amount of food and manufactures consumed, while α represents the food expenditure share. Through standard utility maximization under budget constraint, representative consumers' demand can be shown to be:

$$\frac{\alpha}{1-\alpha}\frac{X_i^c}{X_a^c}=\frac{P_a}{P_i}; \qquad (10.1)$$

where P_a/P_i denotes the agricultural terms of trade. Consistently with the above specification of demand, the price index $\underline{P}$ is

$$\underline{P}=P_a^{\alpha}P_i^{1-\alpha}. \qquad (10.2)$$

10.2.2. Technologies

The agricultural sector produces food employing a backward technology that uses labour and land, but has no scope for reproducible inputs.[10] The agricultural production function is given by

$$X_a^s=A_aL_a^{1-b}; \quad 0\leq b<1 \qquad (10.3)$$

where X_a^s denotes food output, L_a the labour employed in agriculture, $(1-b)$ and A_a are technological parameters describing respectively the degree of returns to labour and the sectoral TFP (which in the case of agriculture summarizes both technological factors but also geographical and climatic conditions). The restriction on b derives from the hypothesis that land endowment is fixed even in the long run,[11] and implies decreasing returns to labour ($b=0$ is a limiting case, representing constant return to labour).

For what concerns the industrial sector, firms utilize labour (in efficiency units) and capital in the production of manufactures. The manufacturing sector is assumed to exhibit increasing returns to scale due to positive external economies stemming from 'capital embodied knowledge' and implying the validity of the Kaldor-Verdoorn law.[12] In other words, we assume that the stock of knowledge is proxied by the average economy-wide stock of capital, and that capital accumulation translates automatically into improvements of the knowledge base and hence of the industrial TFP at the constant rate μ. The present formalization is equivalent to assume a learning-by-doing process, in which the cumulative gross investment represents the index of experience, and knowledge depreciates at the same rate as physical capital.[13]

In accordance with the previous discussion, the industrial technology is described by a Cobb–Douglas production function

$$X_i^s = A_i \tilde{K}^{\mu} K^{\beta} \left(E_{(w_i, w_a)} L_i\right)^{1-\beta}; \; \mu > 0, \; 0 < \beta < 1;$$

where X_i^s, L_i and K denote respectively manufacturing output, industrial labour and capital stock, the function $E_{(w_i, w_a)}$ represents labour efficiency, the parameters β, $(1-\beta)$ and A_i are respectively the capital and labour shares, and the industrial TFP, and finally $\tilde{K}^{\mu}$ represents the external positive effect of capital accumulation, $\tilde{K}$ being the average capital stock of our economy.

The fact that technological economies are *external* to each firm derives from assuming that the non-rival and non-excludable nature of knowledge is such that the experience acquired by one firm spills over *completely* and *immediately* to the others, exerting a positive externality on all manufacturing producers.[14] In light of this, we can argue that in equilibrium the average capital stock of the economy will match that of the representative firm. Accordingly, the industrial production function can be rewritten as

$$X_i^s = A_i K^{\mu+\beta} \left(E_{(w_i, w_a)} L_i\right)^{1-\beta}; \; \mu > 0, \; 0 < \beta < 1. \tag{10.4}$$

Clearly, as long as $\mu > 0$ the above production function displays aggregate increasing returns, though not necessarily constant or increasing returns to capital, as typically assumed in endogenous growth models *à la* Romer or in AK models.[15]

Concluding the analysis of technologies, capital accumulation will clearly not trigger a 'homothetic growth' for the economy as a whole, precisely because in this set-up reproducible inputs are specific to only one sector: industry. Unlike in aggregate models, here the accumulation of reproducible factors affects asymmetrically the marginal productivity of labour in agriculture and manufacturing, leaving the burden of equilibrium adjustment to labour reallocation, capital–labour substitution (in industry) and eventually to price adjustments. At the same time, resource reallocation across sectors determines a change in output composition and employment shares.

10.2.3. Distribution and Labour Market

In line with the traditional literature on dual economies, distributive issues and 'organizational asymmetries' between agriculture and industry play a key role in the present model, especially as concerns the labour market. Our approach, however, departs from the debated hypothesis that wages in the traditional sector are determined *à la* Lewis by the average productivity of labour, giving rise to labour surplus.[16] Instead, we assume perfect competition among rentiers and labourers, such that the former hire all available workers and pay them at a wage rate equal to their marginal revenue product. Analytically we will thus have:

$$W_a = (1-b)\ A_a (L_a)^{-b} P_a; \tag{10.5}$$

and

$$R = b\ A_a\ (L_a)^{1-b} P_a = \frac{b}{1-b}\ W_a\ L_a; \tag{10.6}$$

where W_a represents the rural wage in nominal terms and R the rents.

Organizational dualism comes into play as regards wage determination in the industrial sector, where we assume the existence of an efficiency mechanism, linking labour productivity with the real wage received.[17] In light of such linkage, the problem faced by industrial entrepreneurs will be

$$\max_{L_i, Wi}[\Pi] = A_i\ K^{\mu+\beta}\left(E_{(w_i, w_a)}\ L_i\right)^{1-\beta} P_i - L_i\ W_i;\ \text{subject to } Wi \geq W_a$$

where upper-case W indicates wages in nominal terms (lower-case w are expressed in real terms), and $E_{(w_i, w_a)}$ is a non-decreasing function relating workers' efficiency with the real wage they receive, and with the real wage they could get if working in agriculture. Notably, the problem faced by industrial entrepreneurs is a constrained maximization, since they cannot hire any worker at a wage lower than the reservation wage the latter could get in agriculture.

Consistently with Akerlof's interpretation of labour contracts as partial gift exchanges, the effort function $E_{(w_i, w_a)}$ reflects those sociological considerations (including the real wages paid in the other sector of the economy) that govern the determination of work norms, and hence regulate labour productivity. Suppose, additionally, that the effort function takes the convenient form

$$E(W_i)=\begin{cases} 0 & \text{for } W_i < \omega^{\frac{1}{d}} W_a^{\gamma} \underline{P}^{1-\gamma} \\ \left[\dfrac{W_i/\underline{P}}{(W_i/\underline{P})^{\gamma}}\right]-\omega; & \text{for } W_i \geq \omega^{\frac{1}{d}} W_a^{\gamma} \underline{P}^{1-\gamma} \quad 0<d,\ \gamma<1;\ \omega>0; \end{cases} \tag{10.7}$$

in which the parameter ω implies a minimum threshold to obtain positive effort (see the piecewise definition of the effort function), d is a positive parameter and is lower than one to ensure the effort function is well-behaved (meaning increasing and concave with respect to the real industrial wage $W_i/\underline{P}$), and γ represents the elasticity of industrial real wage to agricultural one. This specification is a generalization of the effort function proposed by Akerlof (1982), and opens the additional possibility of having a less than proportional relationship between the wage received by industrial workers, and the wage they would receive if employed in agriculture.[18]

Under the above assumptions, and as long as the constraint $W_i \geq W_a$ is not binding, the FOC for their profit maximization problem imply the Solow condition of unitary wage elasticity of effort (ensuring cost minimization)

$$W_i = \left(\frac{\omega}{1-d}\right)^{\frac{1}{d}} W_a^{\gamma}\ \underline{P}^{1-\gamma}; \tag{10.8}$$

plus the usual labour demand function

$$L_i = (1-\beta)^{\frac{1}{\beta}}\ A_i^{\frac{1}{\beta}}\ \left(E^*\right)^{\frac{1-\beta}{\beta}}\ K^{\frac{\mu+\beta}{\beta}}\ (W_i)^{-\frac{1}{\beta}}\ P_i^{\frac{1}{\beta}}; \tag{10.9}$$

where $E^* \equiv d\omega/(1-d)$ is the effort level corresponding to W_i. Given that the second order conditions are met for the assumed well-behaving production and effort functions, and that the constraint is satisfied for the assumed values of d, the FOC define the solution of the above profit maximization.

Figure 10.1(a) represents the diagram corresponding to our specification of the effort function on the $W_i - E$ space. The payroll cost per efficiency unit of labour corresponding to each point of the effort function is given by the slope of the ray from the origin to the same point. Clearly, the optimal wage (indicated in the graph as W_i^*) corresponds to the point of tangency between the ray and the effort function, since the said coefficient is at its minimum attainable level.[19] Figure 10.1(b) instead represents the corresponding industrial labour demand on the $W_i - L_i$ space: at W_i^* the labour demand schedule has a kink, because entrepreneurs will resist any wage undercutting and keep the wage at its optimal level. Indeed, wages other than W_i^* would not minimize the cost of labour per efficiency unit and consequently will not be profit maximizing.

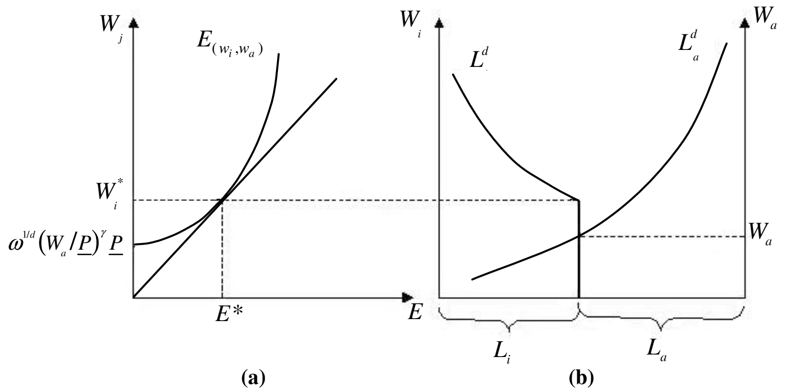

Figure 10.1. The efficiency wage mechanism

Unless the constraint forces them to act differently, capitalists will hence set the wage at W_i^*; as a result of the downward rigidity of the industrial wage, high-earning jobs will be rationed and only L_i^* workers will be hired. The remaining workers will be all employed in the rural sector at the market clearing wage, in accordance with Equation (10.5) (which determines the L_a^d curve in Figure 10.1(b)). Thus a wage gap will arise endogenously across sectors. Clearly, the position of the L_i^d curve depends, among other factors, on the existing stock of capital, with a higher K causing *ceteris paribus* an outwards shift of the curve and hence an increase in L_i.

The adjustment process described so far follows Kaldor's insights according to which employment creation in the manufacturing sector of typical developing countries is constrained by labour demand and not by supply factors.[20] For this reason, the phase in which $W_a < W_i$ and wage gaps arise across economic sectors, will be called hereafter *Kaldorian underemployment*.[21] During this phase,

> a faster rate of increase in the demand for labour in the high-productivity sectors induces a faster rate of labour-transference even when it is attended by *a reduction, and not an increase, in the earnings-differential between the different sectors* (Kaldor, 1968, p. 386, italics in the original).

The complete analytical description of the inputs market during the Kaldorian underemployment phase requires that we derive, in addition to Equations (10.5), (10.6), (10.7), (10.8) and (10.9), total profits and the labour market clearing, which are respectively given by

$$\Pi = \frac{\beta}{1-\beta} W_i L_i; \qquad (10.10)$$

and

$$L_i + L_a = 1. \tag{10.11}$$

Note that in the last equation we have normalized the labour force to 1, such that L_a and L_i respectively represent the employment share of the traditional and of the modern sector; this simplifying normalization, however, comes at the cost of eliminating the effect of demographic variables on our economy.

It should be clear at this point that Kaldorian underemployment persists only as long as the solution implied by the FOC is admissible, that is as long as $W_a < W_i$. Given the hypothesis of diminishing returns to labour in agriculture, however, the withdrawal of labour from the rural sector is bound to increase W_a; moreover, since the elasticity of industrial wages to rural ones is lower than one, eventually the latter will reach W_i and the constraint will become binding. With reference to Figure 10.1(b), the expansion of the industrial sector (a shift of the L_i^d curve toward north-east) tends to close the wage gap, until eventually one uniform wage prevails. Indeed, capitalists are then compelled to pay workers a wage equal to the agricultural one, and the Kaldorian underemployment phase gives way to *economic maturity*: 'a state of affairs where real income per head had reached broadly the same level in the different sectors of the economy'.[22] During the maturity phase employees will be indifferent between working in industry or in agriculture, and thus lack any incentive to increase their effort beyond E^*, despite any possible increase in the uniform real wage rate.

In light of this reasoning, during maturity wages will be set at

$$W_i = W_a; \tag{10.12}$$

while industrial labour demand and profit rate will continue being determined by the same equations holding during Kaldorian underemployment (Equations (10.10) and (10.11)), with the only caveat that now the uniform wage rate replaces the value of W_i determined according to efficiency considerations. Obviously, the rural wage and rent determination, and the labour market clearing will also hold during maturity, so Equations (10.5), (10.6) and (10.11) complement the description of the labour market.

10.2.4. Market Clearing

The complete characterization of the economy involves two more equations related to market clearing for final goods: assuming that the economy is closed to international trade, such conditions are stated directly for food output, and by means of the consumption expenditure flow identity as concerns manufactures. In determining the proportion of income devoted to personal consumption, we also assume that wage income as well as rents are entirely consumed, while profit-earners save a constant proportion s of their total income Π. Our system will therefore be completed by the following two equations:

$$X_a^c = X_a^s; \tag{10.13}$$

for the food market (with the suffixes c and s meaning respectively consumed and supplied), and

$$P_a\ X_a^c + P_i\ X_i^c = W_a\ L_a + R + W_i\ L_i + (1-s)\ \Pi; \tag{10.14}$$

for manufactures.[23] We note in passing that Walras's law can be used to take manufactures as the numeraire, in order to have the industrial product wage equal to its nominal value, such that

$$P_i = 1;\ \underline{P} = P_a^{\alpha}. \tag{10.15}$$

10.2.5. Dynamic of Capital Stock

As concerns the dynamic of the state variable K (hence the long-run characterization of the economy), we follow the Ricardian assumption that savings are automatically reinvested to increase the capital stock. Combining this hypothesis with those underlying Equation (10.14) we can describe the dynamic of the capital stock as

$$\dot{K} = s\Pi - \delta K;$$

where $\dot{K}$ is the time derivative of the capital stock, and δ expresses the depreciation rate of capital. Denoting by $\hat{K}$ the capital growth rate, the dynamic of the capital stock may be rewritten as

$$\hat{K} = s\frac{\Pi}{K} - \delta = sr - \delta; \tag{10.16}$$

where r is the profit rate. This equation represents the fundamental differential equation of our model, and corresponds to the well-known Solow–Swan equation.

Stated as it is, ours is an example of 'supply-limited models of industrial growth' – using Taylor's jargon – with market-clearing prices and flexible capital–labour ratio, as opposed to the fixed prices and technological coefficients characterizing the structuralist literature. It is important to emphasize that the choice of a supply-limited model in this context is not meant to undervalue the importance of Keynesian arguments concerning the level of effective demand, but only to focus our attention on the *potential* growth path of an economy. Apart from the presence of increasing returns in industry, the distinctive feature of this model is the peculiar characterization of the labour market; it is this aspect that permits us to rationalize one crucial insight of the 'dual economy literature': the mismatch, which exists at a low level of development, between the labour productivity in the modern sector and the corresponding opportunity cost of labour in the traditional agricultural sector.

10.2.6. Equilibrium Configuration

Instead of directly solving the whole system of equations and determining the steady states, we prefer to proceed in three stages to highlight the various economic mechanisms at work in the development process. Holding the capital stock as a pre-determined variable – hence in the short run – the economy is analytically described by a system of 14 independent equations with 14 endogenous variables (X_a^c, X_i^c, X_a^s, X_i^s, L_a, L_i, P_a, P_i, $\underline{P}$, W_a, R, W_i, E, Π). It is thus possible to determine the nominal industrial wage consistent with the clearing of the goods' market for each given level of capital stock; hereafter the correspondent locus of short-run equilibria in the $\log W_i - \log K$ space is called the *product wage schedule* (indicated as RW). At a second stage, the *locus of stationary capital stock* can be obtained from the dynamic Equation (10.16), to express the value of the nominal industrial wage (W_i) corresponding to the break-even situation with null net investment. Finally, comparing the relative position of the two loci, the necessary conditions for the existence of steady state equilibria and for their stability properties can be determined on the basis of relative slopes of the two curves. Clearly, because of the dichotomous working of the labour market before and after the maturity threshold $W_a = W_i$, the equilibria shall be derived separately for the two phases.

As emphasized by classical authors (Malthus, Marx and Ricardo above all) and by early development economists of the 1950s and 60s, the elasticity of industrial labour supply is the pivotal magnitude summarizing the economic mechanisms at work. Its crucial role is evident once we note that in two-sector macro models – unlike in aggregate models – this elasticity depends on the interaction between technological conditions (namely the evolution of labour productivity across sectors), demographic variables, and movements in relative prices, while it concurs to determine the speed of labour reallocation from agriculture to industry, and the effect of such a reallocation in terms of profitability.

During Kaldorian underemployment, the 14 equations composing the system are Equations (10.1)–(10.15). From such equations, it can be shown after some algebraic manipulations (see the Mathematical Appendix 10A1.A that the elasticity of labour supply faced by industrial entrepreneurs is equal to

$$\varepsilon^{LS} \equiv \frac{\partial \log L_i}{\partial \log W_i} = \frac{(1-\alpha)\,(1-\gamma)\,(1-L_i)}{\gamma + \alpha(1-\gamma)(1-bL_i)}; \tag{10.17}$$

Two observations are straightforward: labour supply elasticity is non-negative for the assumed parametrization, and is a decreasing function of the food expenditure share α, of the elasticity of industrial wage to that of the agricultural sector γ, and of the industrial employment share L_i. The negative dependency of ε^{LS} on L_i (on α) arises because, *ceteris paribus*, a higher industrial labour share (a higher food expenditure share) turns relative prices in favour of agriculture. Hence the nominal wage W_i will have to grow proportionally more to attract additional workers to industry. The economic reason behind the negative dependency of ε^{LS} on γ lies instead in the fact that, *ceteris paribus*, a higher γ makes industrial wages more sensitive to agricultural ones, such that an increase in L_i triggering a correspondent rise in W_a, will in turn augment industrial nominal wages even faster.

Continuing with a bit of algebra (see the Mathematical Appendix 10A1.B), it can be demonstrated that during Kaldorian underemployment the equation of the short-run equilibrium locus in log terms is given by

$$\frac{\gamma+\alpha(1-\gamma)(1-b)}{\gamma+\alpha(1-\gamma)} \log\left[1-A_i^{\frac{1}{\beta}}(1-\beta)^{\frac{1}{\beta}}\left(E^*\right)^{\frac{1-\beta}{\beta}} \exp\left(-\frac{1}{\beta}\log W_i+\frac{\mu+\beta}{\beta}\log K\right)\right]+$$
$$+\log\left[Q\, A_a^{\frac{\alpha(1-\gamma)}{\gamma+\alpha(1-\gamma)}}\, A_i^{-\frac{1}{\beta}}\right]-\frac{\mu+\beta}{\beta}\log K+\left[\frac{(1-\alpha)(1-\gamma)}{\gamma+\alpha(1-\gamma)}+\frac{1}{\beta}\right]\log W_i=0; \tag{10.18}$$

where Q is a constant defined as

$$Q \equiv \frac{1-\alpha}{\alpha}\frac{(1-\beta)^{-\frac{1-\beta}{\beta}}}{1-s\beta}\left[\frac{(1-d)^{\frac{1}{d}}}{\omega^{\frac{1}{d}}(1-b)^{\gamma}}\right]^{\frac{1}{\gamma+\alpha(1-\gamma)}}\left(E^*\right)^{-\frac{1-\beta}{\beta}}.$$

To determine the long-run equilibrium of the system, instead, one simply needs to replace L_i in Equation (10.16) with its short-run equilibrium value, taken during Kaldorian underemployment from Equation (10.9). This operation, expressing everything in logarithmic terms, yields

$$\log\left[\frac{s\beta}{\delta}(1-\beta)^{\frac{1-\beta}{\beta}}\left(E^*\right)^{\frac{1-\beta}{\beta}}\right]+\frac{1}{\beta}\log A_i-\frac{1-\beta}{\beta}\log W_i^{**}+\frac{\mu}{\beta}\log K=0; \tag{10.19}$$

where we used the notation W_i^{**} in order to distinguish the wage compatible with break-even investment from the short-run equilibrium wage.

Total differentiation of Equation (10.18) yields the coefficient of the product wage schedule, which is equal to

$$\frac{\partial \log W_i}{\partial \log K}=\frac{\mu+\beta}{1+\beta\varepsilon^{LS}}=\frac{(\mu+\beta)\,[\gamma+\alpha(1-\gamma)(1-bL_i)]}{\gamma+\alpha(1-\gamma)(1-bL_i)+\beta(1-\gamma)(1-\alpha)(1-L_i)}. \qquad (10.20)$$

This coefficient is surely positive, given that the labour supply elasticity is non-negative, and furthermore it is decreasing in ε^{LS}. Indeed, a given increase in the capital stock will trigger an outflow of labour from agriculture,[24] and the higher the elasticity of industrial labour supply the smaller – *ceteris paribus* – the adjustment in nominal industrial wages required by the expansion L_i. Besides, since a rise in industrial employment reduces ε^{LS}, the product wage schedule will be flatter for low levels of industrial labour share, and get gradually steeper as industry expands its employment basin. On the other hand, the higher μ, the stronger the external capital effects prompted by the given augment in the capital stock, and the higher the industrial wage in equilibrium; hence the greater the coefficient of the product wage schedule.

As for the stationary capital locus, close inspection of Equation (10.19) shows that in the $\log W_i - \log K$ space it represents a straight line sloped

$$\frac{\partial \log W_i^{**}}{\partial \log K}=\frac{\mu}{1-\beta}. \qquad (10.21)$$

Given the parametrization, the coefficient is positive and increasing in μ: a stronger effect of capital accumulation on the industrial TFP implies higher reinvested profits, and a higher nominal wage compatible with the break-even level of investment. On the other hand, the stationary capital locus is also steeper as the capital share β becomes greater, because a higher β means, *ceteris paribus*, a higher level of total profits for the same increase in capital stock,[25] hence a higher level of reinvestment since the propensity to save is constant.

In plain words, during Kaldorian underemployment higher values of the capital stock trigger the expansion of the industrial labour share and of industrial output, leading the agricultural terms of trade to augment; this relative price movement, summed to the withdrawal of labour from agriculture, causes a sharp rise in the rural wage. Both the relative price movement and the rural wage increase drive the upward adjustment of industrial wages to satisfy the Solow condition. As shown in Mathematical Appendix 10A1.C, the adjustment process required to get the equilibrium in the goods' market is such that higher levels of K entail a reduction in the wage (and productivity) gap between manufacturing and agricultural activities, to the extent that for sufficiently high capital stock a unique uniform (and labour productivity) will prevail in the economy.

Once this happens and the constraint $W_a = W_i$ becomes binding, the system enters the maturity phase and the above equilibrium configuration ceases to hold. Indeed, the short-run characterization of the mature economy is still described by Equations (10.1)–(10.15), but unlike in the Kaldorian underemployment phase Equation (10.12) now replaces Equation (10.8). As shown formally in Mathematical Appendix 10A2.A, the prevalence of one uniform wage significantly alters the dynamic in the labour market: sectoral labour shares stabilize at the constant level

$$\bar{L}_i = \frac{(1-\alpha)(1-\beta)}{(1-\alpha)(1-\beta)+\alpha(1-b)(1-s\beta)}; \; \bar{L}_a = 1-\bar{L}_i; \qquad (10.22)$$

regardless of the capital stock, while the labour supply elasticity turns to zero.[26]

The null elasticity of industrial labour supply during maturity also modifies the product wage schedule, whose equation is then

$$\log\bar{L}_i - \frac{1}{\beta}\log(1-\beta) - \frac{1}{\beta}\log A_i + \frac{1}{\beta}\log W_i - \frac{\mu+\beta}{\beta}\log K - \frac{1-\beta}{\beta}\log E^* = 0; \qquad (10.23)$$

from which we see that the *RW* curve on the usual $\log W_i - \log K$ plane degenerates into a half-line sloped

$$\frac{\partial \log W_i}{\partial \log K} = \mu + \beta; \qquad (10.24)$$

(for a formal proof see the Mathematical Appendix 10A2.B). The slope of the short-run equilibrium locus is now steeper that during the Kaldorian underemployment phase, since the tendency of wages to grow along with capital accumulation (captured by the term $\mu+\beta$) is not mitigated by the effect of elastic labour supply. Considering the whole trend of the RW schedule on the $\log W_i - \log K$ space, it is first increasing and convex as long as Kaldorian underemployment persists, while after the corner point at the maturity threshold it turns into an upward-sloping half line.[27]

As for the long-run equilibrium of the system, given that Equation (10.9) continues to hold even in maturity and that nothing alters the differential equation of capital accumulation, the stationary capital locus will continue to be expressed by the same upward-sloping straight line of Equation (10.19).[28]

Superimposing the short-run and long-run equilibrium loci we can determine the equilibria, at the interception points, and their stability properties, according to the relative position of the two curves. Ideally, the economy moves along the real wage diagram, with the capital stock growing as long as the short-run equilibrium wage lies below the $\hat{K}=0$ locus, and shrinking if the opposite happens. The reason for this is the behaviour of total profits, and hence of investment: when the short-run equilibrium wage lies below that compatible with null net investment, reinvested profits will exceed depreciation costs and fuel capital accumulation, while in the opposite situation net investment will be negative and capital stock will fall. More precisely, it can be shown that

$$\hat{K} = s\ \beta \left[E^{*}\ (1-\beta)\right]^{\frac{1-\beta}{\beta}}\ A_i^{\frac{1}{\beta}}\left(\frac{K^{\mu}}{W_i}\right)^{\frac{1}{\beta}}\ \left(W_i - W_i^{**}\right).$$

Figure 10.2 presents two possible configurations of the system characterized by different parametrizations. A third possible configuration is the one in which the *RW* schedule cuts the stationary capital locus only once and from above. In such a case, for capital stocks lower than that corresponding to the interception of the two curves, the economy is stuck in a low equilibrium trap; while for K higher than the threshold level the system diverges toward an infinite capital stock (and wage rate) with manufacturing production growing indefinitely despite an ultimately stable labour share.[29, 30]

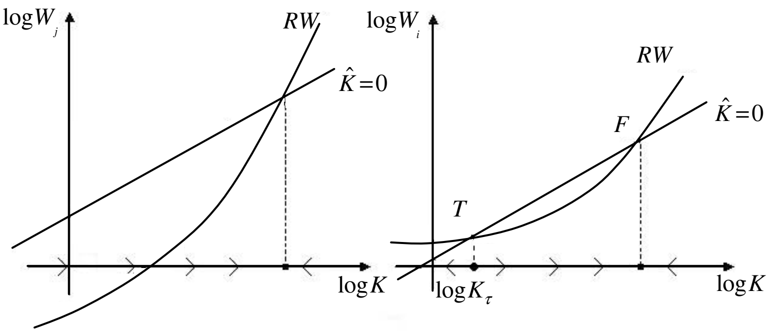

Figure 10.2. The model

Considering at first the case of Figure 10.2, there are two possible equilibria: an unstable equilibrium of pure subsistence at zero capital stock, and an asymptotically stable equilibrium of full-industrialization *F*, where both sectors coexist. The situation depicted resembles closely the results of the standard neoclassical growth model in a two-sector framework. A new feature is, however, the convex interval of the *RW* curve, during which the economy undergoes a process of industrialization (Rostow's take-off), and changes in output and employment composition are fostered by the relatively elastic supply of industrial labour.[31] Moreover, the transference of labour from the low-productive to the high-productive sector entails a double gain in terms of growth: on the one hand, marginal labour productivity in agriculture grows because of diminishing returns to labour (remember that the amount of land is fixed); on the other, increasing returns accelerate the growth of productivity in industry, fuelling the expansion of the capital stock and of the whole manufacturing sector.[32]

The structural dynamic described by the Kaldorian underemployment phase rationalizes several stylized facts often cited in the literature concerning developing countries:[33]

1. the 'agriculture–industry shift', meaning the declining importance of agriculture in terms of both employment share and percentage contribution to GDP in the course of economic development;
2. the existence of wide productivity gaps across economic sectors in developing countries, with agriculture featuring a much higher employment share than its corresponding GDP share, and hence having a lower average labour productivity than the rest of the economy. Such productivity gaps are mirrored by urban–rural wage gaps, which act as a stimulus to labour reallocation toward city-based industrial employment;
3. the progressive reduction of the intersectoral differences in productivity (and wages), as labour reallocation toward industry raises agricultural labour productivity relative to the rest of the economy;
4. the S-shaped dynamic of saving and investment ratios as GDP grows, with a strong acceleration at low–middle-income levels.[34]

Gradually, however, labour supply becomes more and more inelastic, wage gaps close and the system eventually enters the maturity phase and stabilizes its employment structure (see Equation (10.22)) with the coexistence of both sectors. From that point onwards, capital accumulation proceeds at a slower pace, while the combined effect of relative price movements and wage adjustment tends to reduce total profits, bringing the system to the stable equilibrium F.[35] Clearly the maturity stage describes the situation of more developed countries, in which the 'agriculture–industry shift' has already taken place and structural dynamics typically involve the further expansion of the service sector.

Considering Figure 10.2(a), the existence of the equilibrium of full industrialization clearly requires the product wage schedule to be steeper than the stationary capital locus, at least for the maturity phase. Hence, taking the relevant expressions from Equations (10.24) and (10.21), for the existence of the *F* equilibrium it is sufficient that

$$\mu < 1-\beta;$$

meaning that the industrial production function displays decreasing returns to capital.

It is easy to see, in the specific case depicted, that the above inequality is equivalent to the necessary condition for the stability of the *F* equilibrium, such that in the case of Figure 10.2(a) if an equilibrium exists with positive capital stocks, it will certainly be asymptotically stable. On the contrary, had the industrial production function been an AK technology $(\mu = 1-\beta)$ or had it exhibited increasing returns to capital $(\mu > 1-\beta)$, there would have been just one unstable equilibrium at $K=0$, but for any positive value of K capital accumulation could have proceeded indefinitely (in the former case of an *AK* technology) or even at a growing speed (in the latter case).

Alternatively, consider the case illustrated in Figure 10.2(b), where the system displays two interceptions between the short-run and long-run equilibrium schedules. Three equilibria are then possible: (i) a locally stable equilibrium of pure subsistence with zero capital stock, (ii) an unstable low development equilibrium at point T, and (iii) a stable equilibrium of full industrialization at *F*. Since in *T* the product wage schedule cuts the $\hat{K}=0$ locus from above, for capital stocks lower than K_T reinvested profits are insufficient to cover entirely the depreciation, the short-run equilibrium wage being given by the corresponding value of the product wage schedule. There is hence an unstable poverty trap causing capital stock to shrink over time until the economy goes back to the state of pure agricultural subsistence. On the other hand, when $K > K_T$ the effect of increasing returns raises profitability sufficiently to trigger a self-fulfilling process of capital accumulation, driving the system to the equilibrium of full industrialization *F*.[36]

The situation depicted in Figure 10.2(b) may call for a big push *à la* Rosenstein-Rodan,[37] that is a concerted investment capable of bringing the capital stock beyond K_T, breaking the poverty trap and making the industrialization process feasible. The relevance of the big push argument is further reinforced if we consider the role of the 'social overhead capital', and of all sorts of capital characterized by large complementarities, and thus able to crowd in private investments and stimulate significant supply responses.[38]

As in other poverty trap models, given the lack of explicit formulation for the product wage schedule, it is impossible to determine sufficient conditions for the existence of the poverty trap. Nevertheless, we can derive the necessary conditions, which essentially require the short-run equilibrium curve to be flatter than the stationary capital locus. Taking the relevant expressions for the Kaldorian underemployment phase respectively from Equations (10.20) and (10.21), the presence of a poverty trap requires

$$\mu > \frac{1-\beta}{1+\varepsilon^{LS}}; \tag{10.25}$$

where ε^{LS} is valued in the neighbourhood of the point of interception.

In light of the recent wave of criticism against the idea of poverty traps (see Kraay and Raddatz, 2007; and Easterly, 2006) a few words should be spent commenting on the situation described in Figure 10.2(b). First of all, it should be pointed out that the poverty trap discussed here is not driven by lack of savings, but by insufficient profitability. Increases in the saving propensity do not alter the necessary condition for the existence of the poverty trap, but only act as a parametric shift of the two curves, and as such may only change the basins of attraction (see Section 10.3 for more details). As a consequence, the poverty trap may hold even in the presence of international flows of capital, regardless of whether capital markets work perfectly or not. If anything, international capital markets would rather attract resources away from low-yielding national assets, thereby exacerbating the situation.

Secondly, the unstable equilibrium of pure agrarian economy does not necessarily entail a zero growth: the analysis so far has taken sectoral TFP as parameters. However, exogenous technical progress also acts in the agricultural sector, and may spur the growth performances even of a completely agricultural economy (in addition to modifying the whole equilibrium configuration, as will be shown later). Thirdly, it is worth noting that the degree of increasing returns required to make the poverty trap a relevant case in our set-up is far lower than in other aggregate models (see for instance equation 11 of Kraay and Raddat, 2007); even a value of μ around 0.2 (hence within the estimates cited by Kraay and Raddatz, 2007) may be sufficient to make the low equilibrium trap plausible. The reason is that the effect of increasing returns is amplified here by the elasticity of industrial labour supply, a factor rather disregarded in aggregate models of growth, although crucial for classical authors.

Finally, we note in passing that the above model suggests a theoretical mechanism able to link the multiplicity of equilibria with the structural characteristics of the economy, namely the extent of the so-called agriculture–industry shift. Simulated work based on analogous premises (the 'variable returns to scale model') has recently confirmed that this line of reasoning may be empirically fruitful in explaining the poor economic performance of LDCs *vis-à-vis* rich nations (see Graham and Temple, 2005).

10.3. THE EFFECT OF TECHNICAL PROGRESS

10.3.1. The Case of Parametric Increase in Agricultural TFP

So far, our analysis of the two-sector economy has been concerned with the determination of the short- and long-run equilibria abstracting from technical progress, and treating the sector-specific TFPs as exogenous parameters. This approach may be convenient from an analytical point of view, but overlooks one of the main forces – if not the main force – behind the long-term increases in income: technical change.

Needless to say, increases in TFP, be it agricultural or industrial, have an unambiguous positive welfare effect, for they allow a greater supply of goods by using the given amount of resources more efficiently. More complex, however, are the effects of technical progress on the equilibrium configuration for the whole dynamic system. Precisely to grasp these effects, we now carry out some comparative statics exercises with regard to sectoral TFPs.

As seen above, any long-run equilibrium, whether stable or unstable, is basically defined by the system between the relevant expression for the product wage schedule (Equation (10.18) for Kaldorian underemployment and Equation (10.23) for the maturity phase) and the stationary capital locus (Equation (10.19)). To make the notation leaner, let us rewrite the system as

$$\begin{cases} RW(\log W_i,\, \log K,\, A_a,\, A_i) \;=\; 0; \\ \\ G(\log W_i,\, \log K,\, A_a,\, A_i) = 0; \end{cases} \tag{10.26}$$

where the implicit function $RW(.)$ is the short-run equilibrium schedule and $G(.)$ indicates the stationary capital locus.

Besides, recall that the product wage schedule is continuously differentiable with respect to its four arguments (but with the exception of the corner point corresponding to the threshold between Kaldorian underemployment and maturity), while the $\hat{K} = 0$ locus is continuously differentiable on its whole domain, with respect to the four arguments. In light of this, and provided that the Jacobian of system (10.26) is non-singular, the hypotheses underlying the implicit function theorem are satisfied over the whole domain, excluding the neighbourhood of the corner point. With this exception, the implicit function theorem can therefore be applied in the neighbourhood of an equilibrium (call it point *Z*) to rewrite system (10.26) as

$$\begin{cases} RW\left[\log W_i^Z(A_a, A_i), \log K^Z(A_a, A_i), A_a, A_i\right] = 0; \\ G\left[\log W_i^Z(A_a, A_i), \log K^Z(A_a, A_i), A_a, A_i\right] = 0; \end{cases} \tag{10.27}$$

in which $(\log W_i^Z, \log K^Z)$ are the coordinates of the equilibrium point. This formulation of system (10.26) represents the starting point for all comparative statics regarding changes in the sectoral TFPs.

As concerns changes in the agricultural total factor productivity, the chain rule theorem can be used to compute the total derivative of each function in system (10.27) with respect to A_a, obtaining:

$$\begin{cases} \left.\dfrac{\partial RW}{\partial \log W_i}\right|_Z \dfrac{\partial \log W_i}{\partial A_a} + \left.\dfrac{\partial RW}{\partial \log K}\right|_Z \dfrac{\partial \log K^Z}{\partial A_a} = \dfrac{\partial RW}{\partial A_a}; \\ \left.\dfrac{\partial G}{\partial \log W_i}\right|_Z \dfrac{\partial \log W_i^Z}{\partial A_a} + \left.\dfrac{\partial G}{\partial \log K}\right|_Z \dfrac{\partial \log K^Z}{\partial A_a} = \dfrac{\partial G}{\partial A_a}; \end{cases}$$

Solving this last system for $\partial \log W_i^Z / \partial A_a$ and $\partial \log K^Z / \partial A_a$ permits us to obtain, from the sign of these derivatives, the direction in which the new equilibrium value (call it *Z′*) resulting from the change in A_a will lie.

Analytically, it can be shown that[39]

$$\frac{\partial \log W_i^Z}{\partial A_a} = \frac{\begin{vmatrix} -\dfrac{\partial RW}{\partial A_a} & \dfrac{\partial RW}{\partial \log K} \\ -\dfrac{\partial G}{\partial A_a} & \dfrac{\partial G}{\partial \log K} \end{vmatrix}}{|J|}; \quad \frac{\partial \log K^Z}{\partial A_a} = \frac{\begin{vmatrix} \dfrac{\partial RW}{\partial \log W_i} & -\dfrac{\partial RW}{\partial A_a} \\ \dfrac{\partial G}{\partial \log W_i} & -\dfrac{\partial G}{\partial A_a} \end{vmatrix}}{|J|}; \tag{10.28}$$

While these two expressions hold in general over the whole domain (except in the neighbourhood of the corner point), the piecewise nature of the product wage schedule implies that comparative statics should be carried out separately for each phase: Kaldorian underemployment and maturity.

Proceeding with a taxonomic logic, suppose first that the *Z* equilibrium occurs during the Kaldorian underemployment phase. In such a case, the partial derivatives in (10.28) should be replaced with their actual values computed from Equations (10.18) and (10.19). Indicating with J^{KU} the Jacobian corresponding to the Kaldorian underemployment phase, this operation yields:

$$\frac{\partial \log W_i^Z}{\partial A_a} = -\frac{\mu\alpha(1-\gamma)}{[\gamma+\alpha(1-\gamma)]\beta}\frac{1}{A_a\left|J^{KU}\right|}; \frac{\partial \log K^Z}{\partial A_a} = -\frac{\alpha(1-\gamma)(1-\beta)}{[\gamma+\alpha(1-\gamma)]\beta}\frac{1}{A_a\left|J^{KU}\right|}; \quad (10.29)$$

Under the assumed parametrization, (10.29) implies that the two derivatives under consideration assume the opposite sign of $\left|J^{KU}\right|$ (see Mathematical Appendix 10A3.A for more details).

Furthermore, Samuelson's 'correspondence principle between statics and dynamics'[40] can be utilized to prove that

$$\left|J^{KU}\right| > 0 \Leftrightarrow \mu > \frac{1-\beta}{1+\varepsilon^{LS}};$$

meaning that $\left|J^{KU}\right|$ is positive when the corresponding equilibrium point is unstable, and negative in the opposite case.[41]

Moving on to the maturity phase, the same procedure should actually be followed to carry out the comparative statics, replacing the partial derivatives of Equation formula (10.28) with their actual values calculated from Equations (10.23) and (10.19). However, recalling that during maturity both $\partial RW/\partial A_a$ and $\partial G/\partial A_a$ are zero, it can directly be argued that shifts in the TFP of the agricultural sector leave the equilibrium of the mature economy unchanged, regardless of its stability (see Mathematical Appendix 10A3.A for more details).

It is hence demonstrated that:

1. parametric increases in agricultural TFP reduce the basin of attraction of the locally stable equilibrium of pure subsistence, under the condition that an unstable equilibrium in the Kaldorian underemployment phase exists ($\partial \log W_i^Z/\partial A_a$ and $\partial \log K^Z/\partial A_a$ from (10.29) are both negative);
2. alternatively, increases in A_a move the stable equilibrium occurring in the Kaldorian underemployment phase (if any) towards North-East, increasing the steady state value of $\log W_i$ and $\log K$ ($\partial \log W_i^Z/\partial A_a$ and $\partial \log K^Z/\partial A_a$ from (10.29) are both positive);
3. finally, parametric modification of A_a leaves unchanged all equilibria occurring in the maturity phase (if any), since $\partial \log W_i^Z/\partial A_a$ and $\partial \log K^Z/\partial A_a$ from (10.29) are both zero.

These comparative statics results are shown diagrammatically in Figure 10.3, representing the case in which a poverty trap occurs during Kaldorian underemployment (dashed schedules represent the equilibrium loci before the TFP increase).[42] The shift of the product wage schedule (in the Kaldorian underemployment interval), *vis-à-vis* the invariance of the stationary capital locus, reduces the basin of attraction of the low-level equilibrium – from

$(-\infty, \log K^{Z})$ to $(-\infty, \log K^{Z'})$, correspondingly lowering the minimum critical level of capital beyond which increasing returns make industry profitable and capital accumulation self-sustaining. Intuitively, the increase in A_a leads to a larger availability of food for given agricultural employment share and capital stock. This fact lowers the agricultural terms of trade and in turn raises, *ceteris paribus*, the real wages of both sectors, thus allowing higher profitability for capitalist entrepreneurs in industry. Moreover, the increase in A_a helps close the wage gap (remember that $\gamma<1$), thus lowering the level of capital stock at which maturity starts.

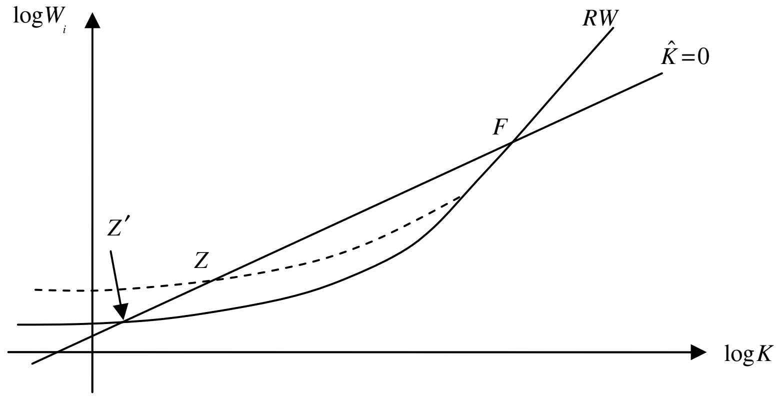

Figure 10.3. The effect of an increase in agriculture TFP

In line with the above results, the importance of the primary sector in the early phases of industrialization is confirmed by the comparison between two emblematic historical cases: the USSR in the 1920s and China in the late 1970s and 80s.[43] In the former case, the land reform of 1917 was unable to stimulate decisive productivity improvements in agriculture, leading to sharp increases in food prices and to great social unrest. As a result, industrialization in USSR entailed deep conflicts between cities and the countryside, and capital accumulation could only take place forcibly. In contrast, China under Deng-Xiao-Ping embarked on a programme of agrarian reforms, which stimulated large productivity improvements. The increase in grain supplies helped maintain urban real wages at a competitive level, favouring capital accumulation and fuelling industrial growth, while the rural sector maintained a reservoir of cheap labour for the high-yielding industrial areas on the coast.

It is important to mention at this stage that the positive link between agricultural productivity growth and industrialization would be further reinforced when including Engel effects in consumers' demand. Nonhomothetic preferences were not used here for lack of explicit solution in the determination of the RW schedule; nevertheless, it is widely acknowledged that Engel effects can play a mayor role in reinforcing structural change, as shown for instance in Murphy et al. (1989), Stokey (1988), Matsuyama (1992), but also in Pasinetti (1993).

Besides, an interesting parallel could be drawn between the role of agriculture in the present model of industrialization and the role of agriculture in the Kaleckian and structuralist interpretation of inflation in developing countries. In the Kaleckian literature, the inability of agricultural productivity to keep pace with the growing industrial sector leads to the so-called 'wage-good constraint': the increase in food prices exerts upward pressure on nominal wages, thus triggering an inflation spiral.[44] In the present set-up, it could be argued that the efficiency wage mechanism during the Kaldorian underemployment phase acts in a way that turns the 'wage-good constraint' into a potential profitability constraint, possibly giving rise to a poverty trap: unless food is available at a sufficiently low price, capital accumulation is simply not self-sustaining for low capital stock, so that the system falls back towards a purely agrarian economy.

10.3.2. The Case of Parametric Increase in Industrial TFP

Applying the same procedure used for parametric changes in the agricultural TFP, we can also shed some light on the comparative statics regarding increases in A_i.[45] Total derivation of system (10.27) with respect to A_i yields

$$\begin{cases} \left.\dfrac{\partial RW}{\partial \log W_i}\right|_Z \dfrac{\partial \log W_i^Z}{\partial A_i} + \left.\dfrac{\partial RW}{\partial \log K}\right|_Z \dfrac{\partial \log K^Z}{\partial A_i} = \dfrac{\partial RW}{\partial A_i}; \\ \\ \left.\dfrac{\partial G}{\partial \log W_i}\right|_Z \dfrac{\partial \log W_i^Z}{\partial A_i} + \left.\dfrac{\partial G}{\partial \log K}\right|_Z \dfrac{\partial \log K^Z}{\partial A_i} = \dfrac{\partial G}{\partial A_i}; \end{cases} \tag{10.30}$$

while solving the above system for $\partial \log W_i^Z / \partial A_i$ and $\partial \log K^Z / \partial A_i$ obtains

$$\frac{\partial \log W_i^Z}{\partial A_i} = \frac{\begin{vmatrix} -\dfrac{\partial RW}{\partial A_i} & \dfrac{\partial RW}{\partial \log K} \\ \\ -\dfrac{\partial G}{\partial A_i} & \dfrac{\partial G}{\partial \log K} \end{vmatrix}}{|J|}; \quad \frac{\partial \log K^Z}{\partial A_i} = \frac{\begin{vmatrix} \dfrac{\partial RW}{\partial \log W_i} & -\dfrac{\partial RW}{\partial A_i} \\ \\ \dfrac{\partial G}{\partial \log W_i} & -\dfrac{\partial G}{\partial A_i} \end{vmatrix}}{|J|}; \tag{10.31}$$

Here again, all partial derivatives should be values at the equilibrium point, and need to be considered separately for Kaldorian underemployment and for maturity because of the piecewise nature of the product wage schedule.

Following a conditional line of reasoning, let us suppose first that the equilibrium Z occurs during the Kaldorian underemployment phase; accordingly, the relevant expressions for the partial derivatives should be computed from Equations (10.18) and (10.19). After some algebra (shown with more detail in Mathematical Appendix 10A3.B) the above formulas reduce to

$$\frac{\partial \log W_i^Z}{\partial A_i} = -\frac{\gamma+\alpha(1-\gamma)(1-bL_i)}{\beta A_i[\gamma+\alpha(1-\gamma)]L_a}\frac{1}{\left|J^{KU}\right|}$$
$$\frac{\partial \log K^Z}{\partial A_i} = -\frac{1-(1-\gamma)[1-\alpha(1-b)]L_i}{\beta A_i[\gamma+\alpha(1-\gamma)]L_a}\frac{1}{\left|J^{KU}\right|}; \tag{10.32}$$

which imply, under the assumed parametrization, that the derivatives $\partial \log W_i^Z / \partial A_i$ and $\partial \log K^Z / \partial A_i$ take the opposite sign of $\left|J^{KU}\right|$. Like in the previous case, the correspondence principle ensures that $\left|J^{KU}\right|$ is positive when Z is an unstable equilibrium, such that the direction in which the new equilibrium lies can be univocally determined.

To complete the conditional analysis, suppose instead that the equilibrium point Z belongs to the maturity interval; in such a case, the relevant partial derivatives in expression (10.31) should be computed from Equations (10.23) and (10.19). After some algebraic manipulation this operation obtains:

$$\frac{\partial \log W_i^Z}{\partial A_i} = -\frac{1}{\beta}\frac{1}{A_i}\frac{1}{\left|J^{MA}\right|};\quad \frac{\partial \log K^Z}{\partial A_i} = -\frac{1}{\beta}\frac{1}{A_i}\frac{1}{\left|J^{MA}\right|}; \tag{10.33}$$

in which J^{MA} indicates the Jacobian corresponding to the maturity interval. Equation (10.33) implies that the derivatives $\partial \log W_i^Z / \partial A_i$ and $\partial \log K^Z / \partial A_i$ take the opposite sign of $\left|J^{MA}\right|$.

In light of the correspondence principle, it can be shown (see Mathematical Appendix 10A3.B) that

$$\left|J^{MA}\right| > 0 \Rightarrow \mu > (1-\beta);$$

such that the sign of $\partial \log W_i^Z / \partial A_i$ and $\partial \log K^Z / \partial A_i$ can be univocally determined.

At this stage, it is therefore possible to summarize the comparative statics results as follows. A parametric increase in A_i

1. shifts the unstable equilibrium (if any) towards the South-West; in more precise terms, it reduces the basin of attraction of the equilibrium of pure subsistence with zero capital stock; this statement holds in both Kaldorian underemployment and maturity, since $\partial \log W_i^Z / \partial A_i$ and $\partial \log K^Z / \partial A_i$ are negative in both cases;
2. moves the stable equilibrium (if any) towards North-East, increasing the steady state level of capital and wages, since $\partial \log W_i^Z / \partial A_i$ and $\partial \log K^Z / \partial A_i$ in this case are both positive.

These results are illustrated graphically in Figure 10.4, which considers the case of existence of the poverty trap (dashed schedules represent the equilibrium loci before the productivity increase).[46] The economic explanation goes as follows, regardless of which phase the economy goes through. The increase in A_i raises, *ceteris paribus*, the supply of manufactures, leading to a moderate increase in the agricultural terms of trade and in agricultural wages, which in turn triggers an upwards adjustment of the nominal industrial wages. These factors explain the upwards move of the *RW* schedule. The rise in industrial productivity brings, however, a much larger gain to entrepreneurs, boosting their profits, and allowing faster capital accumulation; this is reflected in the upwards shift of the stationary capital locus. Since the vertical movement of the $\hat{K} = 0$ locus outweighs that of the product wage schedule[47] the unstable low-development equilibrium (if any) will occur for a lower level of capital stock. Technical progress in industry directly boosts the profitability of entrepreneurs, such that a self-sustaining

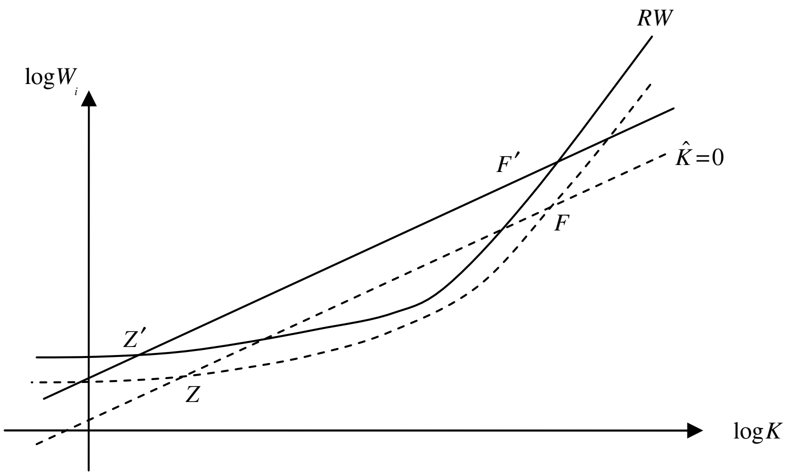

Figure 10.4. The effect of an increase in industrial TFP

accumulation of capital becomes viable even for lower capital stocks. For exactly the same reasons, the equilibrium of full industrialization will always be pushed towards higher levels of capital stock by improvements in industrial TFP, regardless of the phase of the economy.

10.4. CONCLUSIONS

In line with our main objective, we have combined in this two-sector macro-model several aspects emphasized by the neoclassical theory of growth and structural change, with other insights drawn from the more dated literature on dual economies and the big push. Interestingly, the adoption of an efficiency wage mechanism in the urban labour market (unlike in the rural one) and the presence of technological external economies in industry, are sufficient to rationalize a view of the agriculture–industry shift *à la* Kaldor, and to originate possible poverty traps that may justify policies of concerted investment to bring capital stock up to a minimum critical level.

Of course, Kaldor's structure of causality pivots around the central role of effective demand, while we retain a supply-driven framework, resembling in this respect Lewis's model of unlimited supply of labour. Nevertheless, the complex interactions between agriculture and industry, the importance of labour reallocation to the more dynamic sector, and the asymmetric working of the labour market represent common aspects that link the present work to Kaldor's 'Strategic factors in economic development', and highlights the crucial role of industrialization and increasing returns in the process of development.

As concerns instead the long debate on the big push argument, the above analysis has shown how – in the presence of a labour market with a mismatch between the wage and productivity levels in agriculture and industry – even moderate degrees of increasing returns in industry may be sufficient to give rise to poverty traps, since the effect of increasing returns is reinforced by the elastic supply of labour for the more dynamic industrial sector. While this result is encouraging for the plausibility of the mechanisms outlined here, our model is undoubtedly very sensitive to the parametrization adopted and, like the majority of poverty trap models, it 'tends to be lacking in testable quantitative implications'.[48] Nevertheless, we believe the mechanisms analysed here may well be relevant for LDCs,[49] and above all for today's sub-Saharan Africa, the region with the closest conditions to our theoretical framework: an extremely capital-poor agricultural sector and widespread areas of subsistence agriculture.

Besides, this work has shown the peculiar relation between sectoral TFP, and the possible bottlenecks to capital accumulation, explaining how increases in the TFP of either of the two sectors may help make the poverty trap if not less likely at least less stringent. Interestingly, two of the strategies suggested by our model to overcome a poverty trap also appear at the cornerstone of Chinese economic success in the last 20 years: raising agricultural productivity and accumulating physical capital, with special attention to the 'social overhead capital'. Now that these strategies are also becoming the pillar of growing Chinese economic intervention in sub-Saharan Africa, we may have the chance to see these policies at work in those economies with the closest resemblance to our conceptual set-up.

MATHEMATICAL APPENDIX

10A1. The Kaldorian Underemployment Phase

10A1.A. Determination of the labour supply elasticity

Substituting in the consumption expenditure flow identity (Equation (10.14)), the value of agricultural wages and rents (Equations (10.5) and (10.6)), then using the numeraire (Equation (10.15)) and the fact that $\Pi = \beta\, W_i L_i/(1-\beta)$ yields

$$P\, X_a^c + X_i^c = P\, X_a^s + \frac{1-s\beta}{1-\beta}\, W_i L_i;$$

which combined with Equation (10.13) obtains

$$X_i^c = \frac{1-s\beta}{1-\beta}\, W_i L_i;$$

Substituting X_i^c from the demand function (Equation (10.1)), using Equations (10.8) and (10.5) to express the agricultural terms of trade as a function of the nominal industrial wage, and applying the labour market clearing relation and the food production function (Equations (10.11) and (10.13)) yields

$$\frac{1-\alpha}{\alpha}\,\frac{1-\beta}{1-s\beta}\left[\frac{(1-d)^{\frac{1}{d}}}{\omega^{\frac{1}{d}}(1-b)^{\gamma}}\right]^{\frac{1}{\gamma+\alpha(1-\gamma)}} A_a^{\frac{\alpha(1-\gamma)}{\gamma+\alpha(1-\gamma)}}\left(1-L_i\right)^{\frac{\gamma+\alpha(1-\gamma)(1-b)}{\gamma+\alpha(1-\gamma)}} = W_i^{-\frac{(1-\alpha)(1-\gamma)}{\gamma+\alpha(1-\gamma)}}\, L_i; \tag{10A.1}$$

Taking logs obtains

$$\log\left\{\frac{1-\alpha}{\alpha}\frac{1-\beta}{1-s\beta}\left[\frac{(1-d)^{\frac{1}{d}}}{\omega^{\frac{1}{d}}(1-b)^{\gamma}}\right]^{\frac{1}{\gamma+\alpha(1-\gamma)}}\right\}+\frac{\alpha(1-\gamma)}{\gamma+\alpha(1-\gamma)}\log A_a+$$

$$+\frac{(1-\alpha)(1-\gamma)}{\gamma+\alpha(1-\gamma)}\log W_i=-\frac{\gamma+\alpha(1-\gamma)(1-b)}{\gamma+\alpha(1-\gamma)}\log\left[1-\exp\left(\log L_i\right)\right]+\log L_i;$$

from which total differentiation yields the industrial labour supply elasticity as in Equation (10.17).

10A1.B. Determination of the product wage schedule

To determine the product wage schedule, replace L_i in Equation (10A.1) with its value from Equation (10.9), which obtains

$$Q\,A_a^{\frac{\alpha(1-\gamma)}{\gamma+\alpha(1-\gamma)}}\,A_i^{-\frac{1}{\beta}}\,K^{-\frac{\mu+\beta}{\beta}}\,W_i^{\left[\frac{1}{\beta}+\frac{(1-\alpha)(1-\gamma)}{\gamma+\alpha(1-\gamma)}\right]}=$$

$$=\left[1-A_i^{\frac{1}{\beta}}(1-\beta)^{\frac{1}{\beta}}\left(E^*\right)^{\frac{1-\beta}{\beta}}\,W_i^{-\frac{1}{\beta}}\,K^{\frac{\mu+\beta}{\beta}}\right]^{-\frac{\gamma+\alpha(1-\gamma)(1-b)}{\gamma+\alpha(1-\gamma)}};$$

where Q is a constant defined as

$$Q=\frac{1-\alpha}{\alpha}\frac{(1-\beta)^{-\frac{1-\beta}{\beta}}}{1-s\beta}\left(E^*\right)^{-\frac{1-\beta}{\beta}}\left[\frac{(1-d)^{\frac{1}{d}}}{\omega^{\frac{1}{d}}(1-b)^{\gamma}}\right]^{\frac{1}{\gamma+\alpha(1-\gamma)}}.$$

Expressed in log terms, this equation is exactly the RW schedule mentioned in the text (Equation (10.18)).

10A1.C. Evolution of the wage gap

Consider the expression for the agricultural wage relative to the industrial one[50]

$$\frac{W_a}{W_i}=\frac{(1-b)\;A_a\;(L_a)^{-b}\;P_a}{W_i};$$

using Equations (10.8) and (10.5) to express P_a as a function of W_i, and then making use of Equations (10.11) and (10.9), the wage ratio can be rewritten as

$$\frac{W_a}{W_i}=\left[\left(\frac{1-d}{\omega}\right)^{\frac{1}{d}}(1-b)^{\alpha(1-\gamma)}A_a^{\alpha(1-\gamma)}\right]^{\frac{1}{\gamma+\alpha(1-\gamma)}}W_i^{\frac{(1-\alpha)(1-\gamma)}{\gamma+\alpha(1-\gamma)}}$$

$$\left[1-A_i^{\frac{1}{\beta}}(1-\beta)^{\frac{1}{\beta}}\left(E^*\right)^{\frac{1-\beta}{\beta}}W_i^{-\frac{1}{\beta}}K^{\frac{\mu+\beta}{\beta}}\right]^{\frac{-b\alpha(1-\gamma)}{\gamma+\alpha(1-\gamma)}}.$$

Taking logs, the wage ratio becomes

$$\log\frac{W_a}{W_i}=\frac{1}{\gamma+\alpha(1-\gamma)}\log\left[\left(\frac{1-d}{\omega}\right)^{\frac{1}{d}}(1-b)^{\alpha(1-\gamma)}A_a^{\alpha(1-\gamma)}\right]+\frac{(1-\alpha)(1-\gamma)}{\gamma+\alpha(1-\gamma)}\log W_i+$$

$$-\frac{b\alpha(1-\gamma)}{\gamma+\alpha(1-\gamma)}\log\left[1-A_i^{\frac{1}{\beta}}(1-\beta)^{\frac{1}{\beta}}\left(E^*\right)^{\frac{1-\beta}{\beta}}\exp\left(\frac{\mu+\beta}{\beta}\log K-\frac{1}{\beta}\log W_i\right)\right];$$

from which total differentiation yields the following relation:

$$\frac{\partial\log\frac{W_a}{W_i}}{\partial\log K}=\frac{\alpha b(\mu+\beta)(1-\gamma)L_i}{\beta L_a[\gamma+\alpha(1-\gamma)]}+\frac{(1-\gamma)[\beta\,(1-\alpha)L_a-\alpha bL_i]}{\beta L_a[\gamma+\alpha(1-\gamma)]}\frac{\partial\log W_i}{\partial\log K};$$

which is greater than zero, given that the coefficient of the product wage schedule $(\partial\log W_i/\partial\log K)$ is always positive.

Since this derivative is strictly positive for the parametrization assumed above, the wage ratio tends to grow along with increases in the capital stock, from values lower than one (by construction) ultimately reaching one when the system enters the maturity phase and the wage gap disappears. To see this, note that the logarithm is a monotonically increasing transformation of the wage ratio and of the capital stock. Hence the sign of the log-derivative $\partial\log(W_a/W_i)/\partial\log K$ equals the sign of the simple derivative of the wage ratio to capital stock.

10A2. The Maturity Phase

10A2.A. Determination of labour supply elasticity

As before, combining the consumption expenditure flow identity (Equation (10.14)), with wage and rent determination and with food market clearing (Equations (10.5) and (10.12), (10.6) and (10.13)), and using the fact that $\Pi=W_iL_i\beta/(1-\beta)$ yields

$$X_i^c=\frac{1-s\beta}{1-\beta}W_i\;L_i.$$

This relation, combined with Equations (10.1), (10.5), (10.11) and (10.13), obtains

$$(1-L_i)\,\frac{1-\alpha}{\alpha}\,\frac{1}{1-b}=\frac{1-s\beta}{1-\beta}\,L_i;$$

from which we derive

$$\varepsilon^{LS}=0;\;\; \overline{L}_i=\frac{(1-\alpha)(1-\beta)}{(1-\alpha)(1-\beta)+\alpha(1-b)(1-s\beta)}.$$

10A2.B. Determination of the product wage schedule

To obtain the equation of the product wage schedule during the maturity phase, combine the labour demand with the wage determination (respectively Equations (10.9) and (10.12)), and recall that labour efficiency in the maturity phase will still be given by E^*. This yields

$$\overline{L}_i\;L=A_i^{\frac{1}{\beta}}\left(1-\beta\right)^{\frac{1}{\beta}}\left(E^*\right)^{\frac{1-\beta}{\beta}}\;W_i^{-\frac{1}{\beta}}\;K^{\frac{\mu+\beta}{\beta}}.$$

Taking logs obtains from this expression the product wage schedule as given in the text (Equation (10.23)).

10A3. The Effect of Technical Change

10A3.A. Comparative statics: agricultural TFP

From the previous analysis it should be clear that in system (10.27) the relevant equations during Kaldorian underemployment are actually (10.18) for *RW* and (10.19) in place of *G*. Accordingly, the following magnitudes are of interest for comparative statics in the Kaldorian underemployment phase:

$$J^{KU}\equiv\begin{pmatrix}\dfrac{\partial RW}{\partial \log W_i} & \dfrac{\partial RW}{\partial \log K}\\[2ex] \dfrac{\partial G}{\partial \log W_i} & \dfrac{\partial G}{\partial \log K}\end{pmatrix}=$$

$$=\begin{pmatrix}\dfrac{\gamma+(1-\gamma)[\beta(1-\alpha)(1-L_i)+\alpha(1-bL_i)]}{\beta[\gamma+\alpha(1-\gamma)]L_a} & -\dfrac{\mu+\beta}{\beta}\,\dfrac{\gamma+\alpha(1-\gamma)(1-bL_i)}{[\gamma+\alpha(1-\gamma)]L_a}\\[2ex] -\dfrac{1-\beta}{\beta} & \dfrac{\mu}{\beta}\end{pmatrix};$$

and

$$\frac{\partial RW}{\partial A_a}=\frac{\alpha(1-\gamma)}{\gamma+\alpha(1-\gamma)}\frac{1}{A_a};\ \frac{\partial G}{\partial A_a}=0.$$

Replacing the partial derivatives of equation formula with the corresponding values as determined here, obtains after some manipulation (10.29).

As concerns the sign of $\left|J^{KU}\right|$, its direct calculation shows after some algebra that

$$\left|J^{KU}\right|>0\Leftrightarrow\mu>\frac{(1-\beta)[\gamma+\alpha(1-\gamma)(1-bL_i)]}{[\gamma+\alpha(1-\gamma)(1-bL_i)]+(1-\gamma)(1-\alpha)L_a}=\frac{(1-\beta)}{1+\varepsilon^{LS}};$$

which basically verifies the correspondence principle between statics and dynamics.

During maturity, instead, the relevant equations for system (10.27) are Equations (10.23) (for *RW*) and (10.19) (for *G*). Accordingly, we have

$$J^{MA}\equiv\begin{pmatrix}\frac{\partial RW}{\partial\log W_i} & \frac{\partial RW}{\partial\log K}\\ \frac{\partial G}{\partial\log W_i} & \frac{\partial G}{\partial\log K}\end{pmatrix}=\begin{pmatrix}\frac{1}{\beta\ L_a} & -\frac{\mu+\beta}{\beta\ L_a}\\ -\frac{1-\beta}{\beta} & \frac{\mu}{\beta}\end{pmatrix};\qquad\frac{\partial RW}{\partial A_a}=0;\qquad\frac{\partial G}{\partial A_a}=0;$$

Finally, replacing the partial derivatives in equation formula with the corresponding values determined here for the maturity phase, directly obtains

$$\frac{\partial\log W_i^Z}{\partial A_a}=0;\ \frac{\partial\log K^Z}{\partial A_a}=0;$$

proving that the equilibrium occurring in the maturity phase (if any) is invariant to parametric changes in A_a, regardless of its stability properties.

10A2.B. Comparative statics: industrial TFP

Starting with Kaldorian underemployment, the relevant equations for system (10.30) are (10.18) (for *RW*), and (10.19) (for *G*). Hence, in addition to the matrix J^{KU} defined above, the magnitudes of interest for the comparative statics regarding A_i in the Kaldorian underemployment phase are:

$$\frac{\partial RW}{\partial A_i}=-\frac{\gamma+\alpha(1-\gamma)(1-bL_i)}{\beta[\gamma+\alpha(1-\gamma)]L_a}\frac{1}{A_i};\qquad\frac{\partial G}{\partial A_i}=\frac{1}{\beta}\frac{1}{A_i};$$

Replacing these values for the corresponding partial derivatives in Equation (10.31), obtains after some manipulation (10.32). Recalling, finally, the condition for a positive determinant of J^{KU} yields the comparative statics result mentioned in the text.

As concerns the maturity phase, instead, the relevant Jacobian is J^{MA} defined above. From Equations (10.23) for *RW* and (10.19) for *G*, it is possible to compute the magnitudes

$$\frac{\partial RW}{\partial A_i} = -\frac{1}{\beta}\frac{1}{A_i}; \qquad \frac{\partial G}{\partial A_i} = \frac{1}{\beta}\frac{1}{A_i}.$$

Substituting these expressions in Equation (10.31) obtains after a bit of algebra Equation (10.33).

Finally, the direct calculation of $\left|J^{MA}\right|$ verifies the correspondence principle, establishing precisely that

$$\left|J^{MA}\right| > 0 \Leftrightarrow \mu > (1-\beta);$$

and with this last condition the sign of the derivatives $\partial \log W_i^Z / \partial A_i$ and $\partial \log K^Z / \partial A_i$ can be univocally determined.

NOTES

1. In Smith ([1763]1978, p. 579) the author argues: 'That it is easier for a nation, in the same manner as for an individual, to raise itself from a moderate degree of wealth to the highest opulence, than to acquire this moderate degree of wealth'.
2. Authors such as Young (1928), Rosenstein-Rodan (1943) and Nurkse (1953) emphasized the importance of increasing returns, while Nelson (1956), Jorgenson (1961) and – later – Dixit (1973) focused on the role of demographic dynamics in creating poverty traps, in which economic growth in absolute terms is balanced by the counteracting dynamics of population, so that GDP per capita remains at a low level.
3. Notably during the 1950s and 60s only demographic traps had been analysed in mathematical form, while poverty traps based on increasing returns, specialization and pecuniary externalities were treated only in narrative contributions.
4. Pasinetti (1993) states: 'The permanent changes in the absolute levels of basic macro-economic magnitudes are invariably associated with changes in their composition, that is, with the *dynamics* of their *structure*'. (Italics in the original.)
5. Not surprisingly, the empirical literature has found growing evidence of the limited explanatory power of the so-called 'augmented Solow regressions' in the case of poor countries, and has suggested the need to go beyond the common linear specification of the growth process commonly used in cross-country growth regressions. See Durlauf and Johnson (1995); Durlauf et al. (2001 and 2005).
6. Furthermore, the common practice in aggregate models of using a linearly homogeneous production function requires even greater caution, because of the subtle but delicate implications of such restriction in terms of positive theory. Solow himself recognized the difficulty in two different articles: Solow (1956, p. 67) cautions about applying an aggregate production function, which is linearly homogeneous, to the case in which production depends on a 'nonaugmentable resource like land'; Solow (1957, p. 314) states the need to

net out agricultural contribution to GDP when applying the aggregate production function to the analysis of real economies. Given that in poor economies the primary sector features a larger contribution to GDP, it is plausible to expect the limitations intrinsic in the use of aggregate production functions to be more stringent for the analysis of developing countries.

7. As in Ros (2000), the expression 'quasi-one-sector' refers to models where the economy produces both knowledge (an input) and one composite good that can be both consumed or invested.
8. Among the mechanisms proposed to justify the existence of multiple equilibria we may cite: technological non-convexities (see Murphy et al., 1989; Azariadis and Drazen, 1990; Ros and Skott, 1997), saving-based traps with subsistence consumption (see for instance Ros, 2000), learning by doing (see Stokey, 1988; Matsuyama, 2002), credit market imperfections (see Banerjee and Newman, 1993; Galor and Zeira, 1993), and institutional traps (see Murphy et al., 1993).
9. The emphasis on the asymmetries – technological as well as organizational – between agriculture and industry is the distinctive feature of dual economy models, among which Lewis (1954 and 1958), Fei and Ranis (1961), Jorgenson (1961), Preobrazensky (1965), Kaldor (1967 and 1968) and Dixit (1973).
10. The absence of capital among agricultural inputs is evidently inappropriate for high and middle income countries displaying capital-intensive techniques of cultivation. However, it represents a suitable approximation for less developed countries (LDCs). Such an assumption is widely adopted in the literature regarding dual economies; obviously, however, it restricts the relevance of the present model to those countries where subsistence agriculture is especially widespread and the scarce physical capital is employed in non-agricultural activities: above all South Asian and Sub-Saharan African countries.
11. The fixed argument 'land' has been omitted from the production function to make the notation leaner.
12. Concerning technological external economies, see Marshall ([1890]1920) and Scitovsky (1954). Concerning the Kaldor-Verdoorn law, see Verdoorn (1949) and Kaldor (1967, 1968).
13. In this respect, the present model differs from both Arrow's original approach (1962), in which experience is also proxied by cumulative gross investment but without knowledge depreciation, as well as from recent models of structural change that disregard the idea of capital-embodied knowledge and relate the learning process to cumulative output (for instance Krugman, 1987; Stokey, 1988; Matsuyama, 1992 and 2002).
14. Despite the caveats about some more realistic refinements of the learning by doing process, the hypothesis of immediate and complete spillovers is widely used in the literature (see Krugman, 1987; Stokey, 1988; Matsuyama, 1992 and 2002) for it allows us to concentrate on the impact of increasing returns without further analytical complications as regards the market structure.
15. In this way, the formalization of increasing returns overcomes the problem of excessive sensitivity to restrictive parametrization, unlike the whole class of AK models, which necessarily require constant returns to capital. See Stiglitz (1992) and Solow (1994) for a critique of AK models in this respect. Obviously increasing returns to capital arise here only if $\mu > 1-\beta$, with equality yielding constant returns to capital.
16. The labour surplus assumption is followed also by Ros (2000) in his 'Rosenstein-Rodan–Leibenstein model'. This choice, however, ends up postulating the equivalence of rural wage and average labour productivity in agriculture even during the mature phase of the economy. Given that this outcome seems rather hard to justify, we preferred to dismiss the surplus labour assumption in favour of the marginal theory of wages. In any case, maintaining the Lewisian assumption would not change qualitatively the conclusion of our

model, but simply reduce the wage gap across sectors (since the average revenue product exceeds the marginal one in agriculture) and the scope for labour re-allocation towards industry, thus shortening the 'dualistic' phase, as will be clarified below.

17. While efficiency wage mechanisms do not seem appropriate for the rural sector in LDCs, dominated by casual labour and informal relations, they are indeed much more credible for the formal labour markets of the urban industrial sector. This fact was already noted by Mazumdar (1959) and is confirmed by Rosenzweig (1988) and Basu (1997).
18. Note that Akerlof's formalization can be obtained by simply assuming $\gamma = 1$, entailing the perfect proportionality of industrial wages and agricultural ones. Apart from this aspect, the rationality for choosing the above specification is the usual one: the threshold ω is included to avoid the trivial solution of an optimal zero wage (see Akerlof, 1982 for more details), and the restrictions on d are needed to ensure the existence of a unique internal maximum.
19. It should be noted, however, that the effort function depends on the real agricultural wage $(W_a / \underline{P})$ and on the price index $\underline{P}$, so that the optimal industrial wage itself is increasing in W_a and $\underline{P}$.
20. Quoting Kaldor's own words: 'the supply of labour in the high-productivity, high-earning sector is continually in excess of demand, so that the rate of labour-transference from the low to the high-productivity sectors is governed only by the rate of growth of demand for labor in the latter' (1968). See also Kaldor (1967).
21. Kaldor actually calls this situation 'labor surplus', but we preferred a different definition, in order to avoid confusion between the notion applied here, and Lewis's concept of surplus labour. Clearly, the notion of Kaldorian underemployment is logically tied to that of disguised unemployment, but in the present case the mismatch between the shadow wage (that is the opportunity cost of labour outside the modern sector) and the market wage in the industrial sector occurs without any breach of the marginal theory of distribution.
22. The quotation is Kaldor's own definition of economic maturity, which he also defined as 'the end of the dual economy' (1968).
23. We can clarify the reason for closing the model using the consumption flow identity, by making use of some relations explained above: Equations (10.5) and (10.6), together with food market clearing (Equation (10.13)) imply that $W_a L_a + R = P_a\ X_a^s = Pa\ X_a^c$; while Equation (10.10) implies that during Kaldorian underemployment $\Pi = W_i L_i \beta / (1 - \beta)$. Analogous implications hold during maturity, with the only difference that the wage rate is then common across the two sectors for Equation (10.12). Therefore, regardless of the economic phase, Equation (10.14) can be rewritten as $X_i^s - X_i^c = s\Pi$, which shows that in equilibrium the excess supply of manufactures shall equate the total amount of savings of the profit-earners.
24. Industrial labour demand depends positively on the capital stock K (see Equation (10.9)).
25. Recall that $\Pi = W_i L_i \beta / (1 - \beta)$.
26. The economic reason behind this result is the movement of the agricultural terms of trade: under the above preference specification the agricultural terms of trade adjusts in order to maintain the constancy of the expenditure shares; but since the uniform wage is also a linear function of P_a (see Equations (10.5) and (10.12)), in equilibrium the price adjustment will balance out other factors (including capital accumulation) and maintain a stable employment structure.
27. While being a piece-wise function, the short-run equilibrium locus is continuous over its whole domain, and continuously differentiable but with the exception of the corner point.
28. The above statement means that, unlike in the case of the product wage schedule, the stationary capital locus will be represented by a unique linear function on the $\log W_i - \log K$ space, without any corner point.

29. A necessary condition for this case to occur is the presence of constant or increasing returns to capital, as will be confirmed analytically below.
30. Given the lack of explicit solution in the Kaldorian underemployment phase, it is not possible to dismiss from a theoretical point of view a fourth case with no interception between the two schedules, and the short-run equilibrium lying entirely above the stationary capital locus (this necessarily requires decreasing marginal productivity of capital). Comparative statics, however, reveals that such an outcome results from extremely low values of the industrial TFP relative to ω (the threshold level of real wage necessary to obtain positive effort from workers). Clearly, this rather implausible limiting case implies that no matter how big the capital stock, industrialization will never be self-financing, so that the economy will always be stuck at the purely agrarian stage.
31. Population growth, which was omitted from our analysis, would basically prolong the Kaldorian underemployment phase by increasing the number of agricultural workers (since industrial jobs are rationed), and reinforcing the tendency of the labour supply to be elastic.
32. Again Kaldor (1968) expresses this idea very clearly: 'the growth of productivity is accelerated as a result of the transfer at both hands – both at the gaining end and at the losing end; in the first, because, as a result of increasing returns productivity in industry will increase faster, the faster output expands; in the second because when the surplus-sectors lose labour, the productivity of the remainder of the working population is bound to rise'.
33. For a more detailed exposition of these stylized facts see among others Kuznets (1966), Chenery and Syrquin (1975), Syrquin (1989), Taylor (1989) and Bhaduri (1993 and 2003); as regards sectoral wage differential, evidence is often cited in the migration literature, especially for the so-called Todarian models.
34. Note that the S-shaped dynamic of the investment share of GDP may also shed some light on why capital accumulation is a particularly important engine of growth at low- and middle-income levels, while TFP growth becomes the dominant force at high income levels.
35. These findings seem to confirm the empirical evidence which suggests growth accelerations occurring at middle-income level, when capital accumulation is faster and the economy enjoys a double gain from industrialization. See Chenery and Syrquin (1975), who find investment following an S-shaped dynamic, and Syrquin (1985).
36. Again, the stability of F requires diminishing returns to capital. Had the technology displayed increasing returns to K there may have been just one unstable interception, after which the system would diverge indefinitely. This was precisely the third possible configuration we referred to, when commenting on Figure 10.2.
37. According to Ros and Skott (1997): 'the essence of a big-push argument is a model with multiple equilibria in which, under certain initial conditions, the economy gets stuck in a poverty trap that can only be overcome through a "big push". No individual firm may have an incentive to expand on its own, even though the coordinated expansion by all firms will be profitable and welfare enhancing'.
38. In the recent literature, the importance of big push considerations in presence of non-tradeable inputs such as infrastructures (and more generally of social overhead capital) is emphasized also by Ros and Skott (1997) and Sachs (2005). Nevertheless, the relevance of possible coordination failures requiring a minimum critical effort should not necessarily lead to the so-called 'classical aid narrative' (see Easterly, 2006), which claims that a sufficient amount of aid would automatically lift countries out of the poverty trap to the take off.
39. Of course in the following section all partial derivatives should be valued at Z, that is at the value corresponding to the equilibrium; for simplicity we omit this detail from the notation of the text.

40. The principle was analysed by Samuelson in 1941; for a recent treatment of the principle see Gandolfo (1997).
41. Recall that for the implicit function theorem to hold, $\left|J^{KU}\right|$ must be different from zero.
42. Clearly, in the absence of the poverty trap the only significant effects of agricultural technical progress concern the possibility that the full industrialization equilibrium – if occurring in the Kaldorian underemployment phase – is pushed towards North-East, with higher steady state levels of capital stock and industrial wages.
43. In this respect more recent evidence concerns the contrasting experience of Asian and African countries as regards the impact of the Green Revolution in raising agricultural yields (see Sachs, 2005). Whereas in the former countries agricultural productivity rose steadily during the 1970s paving the way for subsequent industrialization, in sub-Saharan Africa food production per capita actually fell. Though attractive, the picture in this case is however blurred by other factors such as demographic changes, soil depletion, desertification and so on.
44. See for instance Kalecki (1976), Dutta (1988) and Basu (1997). Note however that this idea was already present in Kaldor (1967), with special reference to the burst of inflation crisis in Latin America.
45. Note that, because of the algebraic properties of Cobb–Douglas production functions, all forms of technical change – unbiased, labour augmenting and capital augmenting (also called Hicks neutral, Harrod neutral and Solow neutral) – translate into variations of the parameter A_i, and are thus essentially indistinguishable from one another.
46. In the absence of a poverty trap, the only impact of the industrial productivity increase would be to increase the steady state level of capital and wages for the stable interception.
47. This can be verified by directly computing $\partial \log W_i / \partial A_i$ for the product wage schedule and for the stationary capital locus: this derivative in the latter case outweighs the correspondent derivative for *RW*.
48. The quotation is taken from Azariadis and Stachurski (2005), recognizing a limit which is common to most poverty trap models..
49. In this respect, our judgement seems confirmed by the simulations presented in Graham and Temple (2005), which suggest that the presence of multiple equilibria may play a key role in explaining income dispersion across countries and be particularly suitable to characterize the poorest LDCs.
50. Note that the absolute wage gap is tied to the wage ratio by the following relation $W_i - W_a = (1 - W_a / W_i)\ W_i$.

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