1. Footloose capital and productive public services

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1.1. INTRODUCTION

According to Brakman et al. (2005), European Cohesion Policy is inconsistent since it sometimes seems to target agglomerations of industrial activities in core regions, but more often stimulates their relocation in the periphery. Such a criticism provides a motivation to analyse policy issues in New Economic Geography (NEG) models, which mainly focus on the determinants of the spatial location of the manufacturing industry.[1] However, incorporation of public expenditure in NEG models represents a very recent theoretical advance. Providing a full picture of the working of both agglomeration and dispersion forces induced by policy measures to make backward regions more attractive to investors might be helpful to design effective policies promoting a sustained catch-up process across EU regions.

The impact of public expenditure on the location decisions of firms has been studied within a few variants of NEG models.[2] In particular, in a companion paper (Commendatore et al., 2008), we consider a two-region Constructed Capital (CC) model, that is a Footloose Capital (FC) model with the additional feature of creation and destruction/depreciation of capital goods. In that paper our main focus was on industrial location and welfare effects of productive public services provision under the assumption of endogenous capital. The government uses tax revenues to purchase capital goods to use in the production of freely available public services. Hence, public policy can affect production in the manufacturing sector via its impact on factor productivity. We show in that paper that the interplay of two effects determines the final impact of an increase in productive public spending in the backward region on the spatial distribution of firms: the productivity effect and the demand effect. On the one hand, an increase in the provision of public services in one region lowers labour input requirements and leads firms to relocate there (productivity effect); on the other, the increase in taxation required to finance this provision and the consequent contraction in private expenditure for manufactures favour dispersion via a change in the relative market size (demand effect). As a result, whether or not higher provision of public services leads firms to relocate in the backward region will depend upon the financing scheme of public expenditure. It is shown that if the burden of taxation falls in a larger proportion on the backward region, the reduction in its relative market size – by making that region less attractive to manufacturing firms – could make the increase in labour productivity ineffective. However, the demand effect can be made negligible[3] if the government makes each region's taxpayers contribute on the basis of their contribution capacity, leaving the relative market size unaltered.

In this chapter we provide further insights into how the interplay of the above-mentioned productivity and demand effects influences industrial location. In order to do so, we consider a much simpler analytical framework, which allows full characterization of the dynamic behaviour of capital movements in response to variations in the degree of trade freeness and in the provision of public services.

We deal with an FC model without an investment sector. In the standard Core–Periphery (CP) model (Krugman, 1991) it is assumed that mobile workers spend their incomes locally and imperfectly competitive firms tend to locate in the biggest market, enforcing the so-called 'home market' effect which leads to agglomeration. The 'cost of living' effect – goods are cheaper in regions with higher concentrations of industrial firms – also favours agglomeration. On the other hand, the market-crowding or competition effect – the presence of an increasing number of competing firms lowers retail sales and wages – favours dispersion. The standard result of the model is that, at a sufficiently low level of trade costs – the so-called 'break point' – when a migration shock – driven by real wage differentials – occurs, agglomeration forces outweigh those of dispersion in such a way as to end up with all industry being concentrated in one region. As a result, complete agglomeration in one region is a stable equilibrium.

The FC model departs from the CP model by virtue of two crucial assumptions: a fixed capital requirement for each variety of the differentiated good and workers' international immobility. In contrast with Krugman (1991), the mobile factor (capital) earnings are repatriated and spent where the capital owner resides. Therefore the typical CP feature of demand-linked circular causality – production changes brought about by factor movements yield expenditure-switching that in turn generates further production changes – does not arise. Furthermore, since costs of living are irrelevant to the production location decisions of capitalists, the cost-linked circular causality of the CP model – shifts in production alter prices inducing worker migration with further production shifting – is eliminated. Hence, the CP outcome of catastrophic agglomeration in one region is ruled out. However, agglomeration still occurs due to the working of the home market effect.

As far as policy is concerned, we assume that the government decides on the levels of the public services in both regions and on the contribution of the two regions to its financing (which may not be proportional to the amount of public services supplied to the region). We assume that public services are produced by using agricultural commodities and increase productivity in the local manufacturing sector, not having any impact on private utility.

Extending Commendatore et al. (2008), we focus on the dynamic structure of the model. First, we fully characterize the dynamic process underlying capital movements and analyse the long-run equilibrium given as a fixed point of the capital mobility dynamics for different degrees of trade freeness. Second, we study the impact of the provision of public services on the long-term behaviour of the regional shares of capital.

The chapter is structured as follows. Section 1.2 provides an empirical background to recent trends of public investment and critical factors influencing foreign direct investment (FDI) in European regions. In Section 1.3 we introduce the assumption of our model; Section 1.4 presents the full model short-run structure, explicitly specifying the capital migration process. Section 1.5 deals with the dynamics of the model, studying the stability properties of core–periphery and interior equilibria, while Section 1.6 presents the decomposition of the final impact of the provision of public services on the long-run allocation of capital across regions into productivity and demand effects. We run such a policy experiment under different degrees of economic integration. Finally, Section 1.7 summarizes and concludes.

1.2. EMPIRICAL BACKGROUND

In order to enhance the attractiveness of backward regions, European regional policy relies heavily on public investment,[4] viewed as 'contributing directly to economic growth and strengthening the productive potential of the economy'. Public investment includes investment in human capital and infrastructure as well as expenditure on education and training aimed at improving the skills of the work force.

As documented by the European Commission (2007), in recent years public investment in EU countries has experienced a declining path at the national level.[5] The share of public investment to GDP in the EU15 declined from 2.9 per cent in 1993 to 2.4 per cent in 2005. The main factors explaining this declining pattern are 'the general tendency towards a shrinking public sector, the increased involvement of the private sector in public sector capital projects and the pressure to reduce overall public expenditure to comply with rules on the budget deficit'.

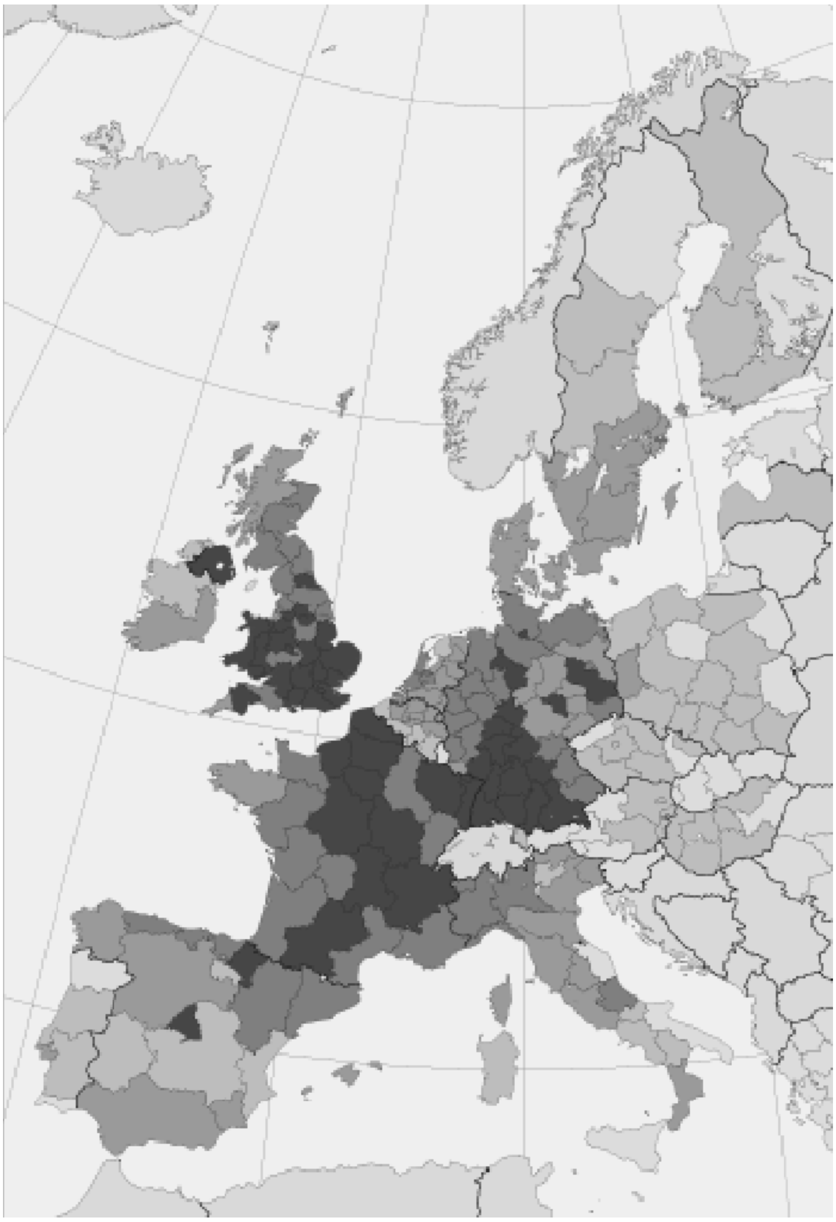
The management of public investment has in fact gradually been decentralized to regional and local levels. As a result, the share of regional and local authorities in public investment has increased in average in all EU countries, underlying the greater responsibility of local governments in the allocation of development-related public spending.

In the view of the European policy maker, FDI flows are interpreted as a critical factor for the development of lagging regions, particularly for the new Member States, representing the 'primary way in which the productivity gap between the industries and services located there and those in the rest of EU can be narrowed' (European Commission, 2007, p. 73). Policies are hence targeted to enhance the attractiveness of regions via the improvement of basic and ICT infrastructures, and education of the work force. However, what we refer to as the demand effect induced by public spending – public expenditure changes imply changes in regional market size – seems to be neglected by the policy maker according to whom 'policy cannot affect factors such as national market size' (European Commission, 2007, p. 74). The neglecting of such an issue provides further motivation for our study.

Data on FDI account for a heterogeneous set of transactions, only part of which relates to actual location decisions. However, the scarcity of available alternative information makes them a natural proxy for capital movements across borders (Buettner, 2002).

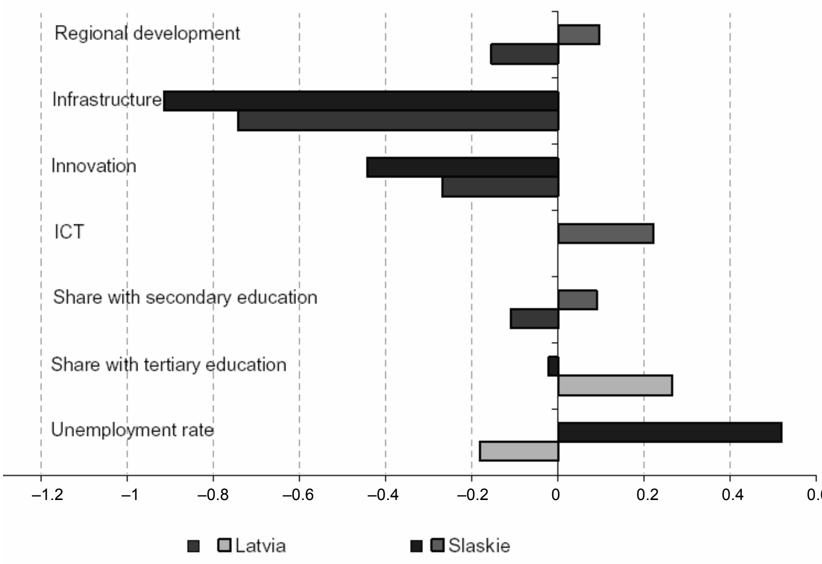
FDI intensity – measured in terms of employees in foreign-owned firms in relation to the total number of employees – across European regions is shown in Figure 1.1. Regions with the largest shares are concentrated in the UK, Germany and France, while Spain has only two regions with a large share (Madrid and Navarra). The regions bordering France and the Atlantic also tend to have larger than average shares. The highest concentration is reached in Randstad regions, in Belgium, in Brussels and most of the Flemish regions and in Ireland (in the regions in which Dublin and Cork are situated). In contrast, all the new Member States, Finland, Greece, Portugal and southern Italy have below average shares.

Figures 1.2–1.5 report the estimated relative importance of factors that can make backward European regions more attractive to foreign investors (Copenhagen Economics, 2006). The figures refer to four subsets of regions: 'Eastern Europe', 'cohesion countries', 'regions facing weaknesses in competitiveness and employment' and 'remote regions'. In each case, figures are constructed for the potentially most attractive regions within the respective subset. The common pattern is that even the better endowed



Note: The darker the areas, the higher the intensity of FDI
Source: Copenhagen Economics (2006)

Figure 1.1. FDI intensity in 2004 (employees in foreign firms as percentage of total number of employees)



Note: The regional attraction factors are reported as the deviation from the regional average in Eastern Europe divided by the regional average in Eastern Europe. Positive values represent a better situation than the EU27 average whereas negative values depict a less attractive situation. The opposite applies to the unemployment rate: negative values represent a better situation than the EU27 average whereas positive values depict a worse situation. ICT data for Latvia are not available.

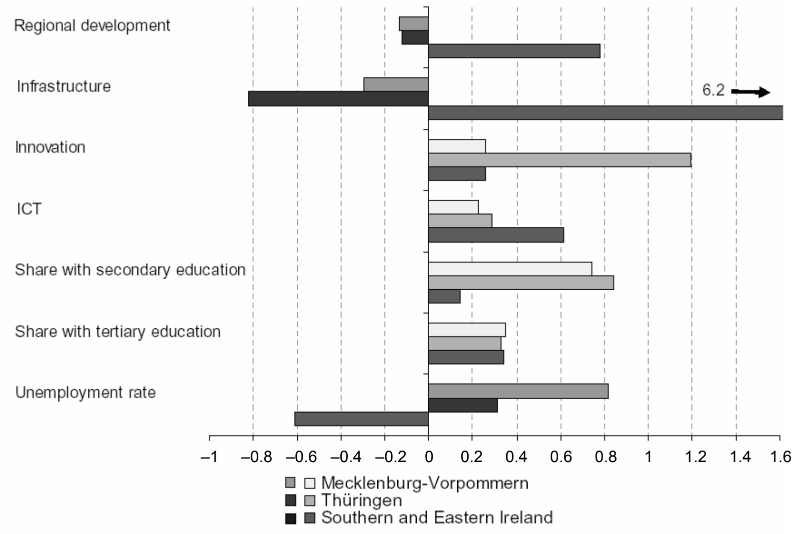
Source: Copenhagen Economics (2006)

Figure 1.2. Regional attraction factors of Latvia and Slaskie relative to the regional average in Eastern Europe

regions in each subset still suffer from major weaknesses with respect to the EU27 average, especially with respect to infrastructure and human capital. This suggests that there is great scope for European regional policy to reduce regional disparities in backward regions via policies which have a local impact but are financed at a more central level.

1.3. ASSUMPTIONS

The Footloose Capital model involves two countries or regions, $r = 1,2$, each with a monopolistically competitive manufacturing sector and a perfectly competitive agricultural sector. There are, in total, L consumers with a share s_L located in region 1. Each consumer provides one unit of labour per



Note: The regional attraction factors are reported as the deviation from the regional average of the cohesion countries divided by the regional average in the cohesion countries. Positive values represent a better situation than the EU27 average whereas negative values depict a less attractive situation. The opposite applies to the unemployment rate: negative values represent a better situation than the EU27 average whereas positive values depict a worse situation.

Source: Copenhagen Economics (2006)

Figure 1.3. Regional attraction factors of Mecklenburg-Vorpommern, Thüringen and Southern and Eastern Ireland relative to the average of regions in the cohesion countries

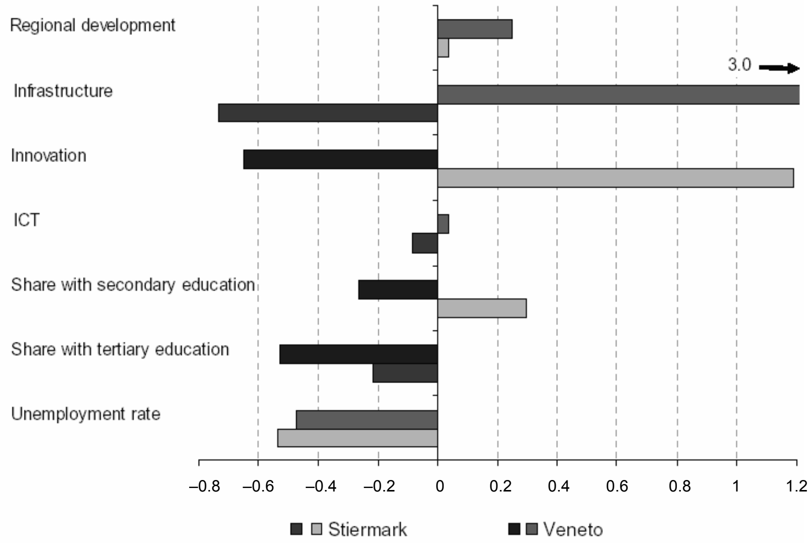
period. Labour is immobile between regions but instantaneously mobile between sectors. Consumers are also the capital owners.

A key feature of the Footloose Capital model is that physical capital is mobile between regions but capital owners are completely immobile and they spend all their earnings in the region in which they live. Consequently, our notation must differentiate between regional shares in capital ownership: s_K is the share of physical capital owned by capitalists resident in region 1 and λ_t is the share of physical capital located and used in region 1 in period *t*.

A representative consumer has the following utility function:

$$U=\left(C_A\right)^{1-\mu}\left(C_M\right)^{\mu}. \qquad (1.1)$$

Utility depends in the usual form on the quantity consumed of a homogeneous agricultural good, C_A, and on a quantity index C_M that is a



Note: The regional attraction factors are reported as the deviation from the regional average in the Cohesion countries divided by the regional average in the Cohesion countries. Positive values represent a better situation than the EU27 average whereas negative values depict a less attractive situation. The opposite applies to the unemployment rate: negative values represent a better situation than the EU27 average whereas positive values depict a worse situation.

Source: Copenhagen Economics (2006)

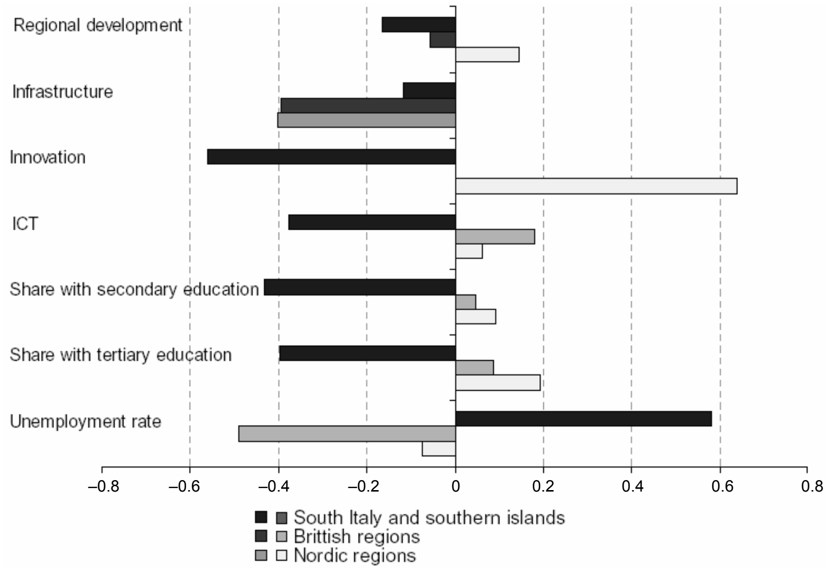
Figure 1.4. Regional attraction factors of Veneto and Steiermark relative to the average of regions facing weaknesses in competitiveness and employment

CES function of the varieties of manufactured goods. The constant elasticity of substitution between the manufactured varieties is denoted by $\sigma>1$; the lower is σ, the greater the consumers' taste for variety. The exponents of the agricultural good and of the manufacturing composite in the common utility function – $(1-\mu)$ and μ, respectively – indicate the invariant shares of disposable income devoted to the agricultural good and to manufactures; therefore $0<\mu<1$.

We have a government that provides public services for the two regions H_r, which increase productivity in the regional manufacturing sector. For one unit of the public service one unit of the agricultural commodity is used; the respective production function is:

$$H=C_{AG}. \tag{1.2}$$

Public expenditures are financed by income tax. While tax rates may differ between regions, the overall budget is always balanced.



Note: The regional attraction factors are reported as the deviation from the EU27 regional average divided by the EU27 regional average. Positive values represent a better situation than the EU27 average whereas negative values depict a less attractive situation. The opposite applies to the unemployment rate. The opposite applies to the unemployment rate: negative values represent a better situation than the EU27 average whereas positive values depict a worse situation. Innovation data for the remote British regions are not available.

Source: Copenhagen Economics (2006)

Figure 1.5. Regional attraction factors of three groups of remote regions relative to the EU27 average

The agricultural commodity is produced with labour as the sole input, one unit of labour yielding one unit of the agricultural product. We assume that neither region has enough labour to satisfy the total demand of both regions for the agricultural good. Thus, both regions always produce the agricultural commodity – the so-called non-full-specialization condition. Transportation of the agricultural product between regions is costless.

Manufacturing involves increasing returns: each manufacturer requires a fixed input of 1 unit of capital to operate and has a marginal labour requirement $\beta_r = f(H_r)$ that depends (negatively) upon the locally provided public services, where $f' < 0$ and $f'' > 0$. A very simple function which satisfies these properties is

$$f(H_r) = A(1 + BH_r)^{-1},$$

where A and B are positive constants.

Transport costs for manufactures take an iceberg form: if 1 unit is shipped between the regions, $1/T$ arrives where $T \geq 1$. Following Baldwin et al. (2003), to compact the notation, we introduce the parameter $\phi \equiv T^{1-\sigma}$ which is conventionally labelled 'Trade Freeness', where $0 < \phi \leq 1$, with $\phi = 1$ corresponding to no trade cost ($T = 1$) and with $\phi \to 0$ corresponding to trade cost becoming prohibitive ($T \to \infty$). The manufacturing sector involves Dixit–Stiglitz monopolistic competition. Given the consumers' preference for variety, a firm would always produce a variety different from the varieties produced by other firms. Thus the number of varieties is always the same as the number of firms. Furthermore, since one unit of capital is required for each manufacturing firm, the total number of firms/varieties, n, is always equal to the total supply of capital:

$$n = K \tag{1.3}$$

The number of varieties produced in period t in region r is:

$$n_{1,t} = \lambda_t n = \lambda_t K \qquad n_{2,t} = (1-\lambda_t)n = (1-\lambda_t)K \tag{1.4}$$

As with most economic geography models, the primary focus of the Footloose Capital model is the spatial location of the manufacturing industry, governed here by the endogenous regional allocation of capital, λ_t, where $0 \leq \lambda_t \leq 1$.

In what follows, we complete the model by characterizing the short-run general equilibrium in period t contingent on λ_t, by specifying explicitly the capital migration process, and by analysing the long-run equilibrium given as a fixed point of capital mobility dynamics.

1.4. SHORT-RUN GENERAL EQUILIBRIUM

With the instantaneous establishment of equilibrium in the agricultural market and no transport costs, the agricultural price is the same in both regions. Since competition results in zero agricultural profits, the equilibrium nominal wage of workers in period t is equal to the agricultural product price and is therefore always the same in both regions. We take this wage/agricultural price as the *numeraire*. Under the assumption of identical behaviour, each firm sets the same local (mill) price p_r using the Dixit–Stiglitz pricing rule. Given that the wage is 1, the local price of every variety is:

$$p_r = \frac{\beta_r \sigma}{\sigma - 1} \qquad p_1 = p \qquad p_2 = p\frac{\beta_2(H_2)}{\beta_1(H_1)} = ph \tag{1.5}$$

where $h \equiv \beta_2 / \beta_1$.

The effective price paid by consumers for one unit of a variety produced in the other region is $p_r T$.

Short-run general equilibrium in period t requires that each manufacturer meets the demand for its variety.[6] For a variety produced in region r:

$$q_{r,t} = d_{r,t} \tag{1.6}$$

where $q_{r,t}$ is the output of each manufacturer in region r and $d_{r,t}$ is the demand for that manufacturer's variety. From Equation (1.5), the short-run equilibrium profit per variety in region r is:

$$\pi_{r,t} = p_r q_{r,t} - \beta_r q_{r,t} = \frac{p_r q_{r,t}}{\sigma} = \left[\frac{\beta_r}{\sigma - 1}\right] q_{r,t} \tag{1.7}$$

This profit per variety constitutes the regional rental per unit of capital.

Consumers face regional manufacturing price indices given by:

$$\begin{aligned} G_{1,t} &= \left[n_{1,t} p_1^{1-\sigma} + n_{2,t} p_2^{1-\sigma} T^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = \left[\lambda_t + (1-\lambda_t)\phi z\right]^{\frac{1}{1-\sigma}} K^{\frac{1}{1-\sigma}} p \\ G_{2,t} &= \left[n_{1,t} p_1^{1-\sigma} T^{1-\sigma} + n_{2,t} p_2^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = \left[\lambda_t \phi + (1-\lambda_t) z\right]^{\frac{1}{1-\sigma}} K^{\frac{1}{1-\sigma}} p \end{aligned} \tag{1.8}$$

where $z \equiv h^{1-\sigma}$ in order to simplify the notation. Consumption per variety in each region is:

$$\begin{aligned} d_{1,t} &= \left[M_1 G_{1,t}^{\sigma-1} + M_2 G_{2,t}^{\sigma-1} \phi\right] p_{1,t}^{-\sigma} \\ d_{2,t} &= \left[M_1 G_{1,t}^{\sigma-1} \phi + M_2 G_{2,t}^{\sigma-1}\right] p_{2,t}^{-\sigma} \end{aligned} . \tag{1.9}$$

M_r denotes the expenditure on manufactured goods in region r; M defines the world expenditure on manufactures $M = M_1 + M_2$ and $s_E = M_1 / M$ its regional split. We see below that M_r, M and s_E are independent of λ_t. From Equations (1.6), (1.8) and (1.9)

$$\begin{aligned} q_{1,t} = d_{1,t} &= \left[\frac{s_E}{\lambda_t + (1-\lambda_t)\phi z} + \frac{(1-s_E)\phi}{\lambda_t \phi + (1-\lambda_t) z}\right] \frac{1}{p} \frac{M}{K} \\ q_{2,t} = d_{2,t} &= \left[\frac{s_E \phi}{\lambda_t + (1-\lambda_t)\phi z} + \frac{1-s_E}{\lambda_t \phi + (1-\lambda_t) z}\right] \frac{1}{p z^{\frac{\sigma}{1-\sigma}}} \frac{M}{K} \end{aligned} \tag{1.10}$$

Therefore – see Equation (1.7) – short-run equilibrium profit per variety in region r is:

$$\pi_{1,t}=\left[\frac{s_{E,t}}{\lambda_t+(1-\lambda_t)\phi z}+\frac{(1-s_{E,t})\phi}{\lambda_t\phi+(1-\lambda_t)z}\right]\frac{1}{\sigma}\frac{M}{K}$$
$$\pi_{2,t}=\left[\frac{s_{E,t}\phi}{\lambda_t+(1-\lambda_t)\phi z}+\frac{1-s_{E,t}}{\lambda_t\phi+(1-\lambda_t)z}\right]\frac{z}{\sigma}\frac{M}{K} \qquad (1.11)$$

For future reference, note that regional and world profit incomes, $\Pi_{r,t}$ and Π respectively, are given by

$$\Pi_{1,t}=\lambda_t K\pi_{1,t} \quad \Pi_{2,t}=(1-\lambda_t)K\pi_{2,t} \quad \Pi=\Pi_{1,t}+\Pi_{2,t}=\frac{M}{\sigma} \qquad (1.12)$$

(for the latter use Equation (1.11)) and world gross income Y by

$$Y=L+\frac{1}{\sigma}M\ . \qquad (1.13)$$

The government plans a total level of public services of H. H_r is the fraction of public services allocated to region *r*. Providing one unit of *H* costs 1 (the agricultural commodity is the numeraire). Total public expenditure is

$$PE=PE_1+PE_2=H_1+H_2\ . \qquad (1.14)$$

The government also decides upon the regional contribution to public service financing, with s_F denoting the share of region 1. Therefore the tax burdens of the two regions are

$$TB_1=s_F H \qquad TB_2=(1-s_F)H \qquad (1.15)$$

Regional expenditure on manufactured goods is therefore given as

$$M_1=\mu(s_L L+s_K\Pi-TB_1) \qquad M_2=\mu[(1-s_L)L+(1-s_K)\Pi-TB_2] \qquad (1.16)$$

World expenditure on manufacture is

$$M=\mu(L+\Pi-H)=\mu\left(L+\frac{M}{\sigma}-H\right). \qquad (1.17)$$

Therefore,

$$M=\frac{\mu}{\sigma-\mu}\sigma(L-H)\ . \qquad (1.18)$$

Its regional split is

$$s_E = \frac{\left[s_L(\sigma-\mu)+s_K\mu\right]L-\left[s_F(\sigma-\mu)+s_K\mu\right]H}{\sigma(L-H)}. \qquad (1.19)$$

World income (see Equation (1.13)), total expenditure on manufactures and its regional split are constant, that is independent of the regional allocation of capital.

With no provision of public services and no taxation, $H = 0$, region 1's share of total expenditure s_E is equal to

$$\widetilde{s_E} \equiv s_L \frac{\sigma-\mu}{\sigma} + s_K \frac{\mu}{\sigma}$$

When $\widetilde{s_E} \neq 1/2$ factor endowments are unevenly distributed between the regions. In particular, when $\widetilde{s_E} < 1/2$ region 1 is poorer (has a smaller factor endowment) than region 2.

With the provision of public services, $H > 0$:

$$s_E > \widetilde{s_E} \quad \text{if} \quad s_F < s_L. \qquad (1.20)$$

For region 1, the expenditure share for manufactured goods after taxation s_E will be greater than the expenditure share for manufactured goods before taxation $\widetilde{s_E}$, if consumers in region 1 contribute less than consumers in region 2 to financing public services. If $s_F = s_L$ the expenditure share for manufactured goods is not affected by taxation.

Finally, Equations (1.11), (1.18) and (1.19) result in short-run equilibrium regional profits per variety.

Crucial for the subsequent dynamics is the relative profitability of capital $R(\lambda_t) = \pi_{1,t}/\pi_{2,t}$ given by:

$$R(\lambda_t) = \frac{1}{z} \frac{\left[\lambda_t\phi+(1-\lambda_t)z\right]+(1-s_E)\phi\left[\lambda_t+(1-\lambda_t)\phi z\right]}{s_E\phi\left[\lambda_t\phi+(1-\lambda_t)z\right]+(1-s_E)\left[\lambda_t+(1-\lambda_t)\phi z\right]}. \qquad (1.21)$$

For a constant s_E the relative profitability of capital depends upon the allocation of capital λ_t via the so-called 'competition effect': a higher λ_t increases the competition in region 1 and therefore reduces relative profitability. The competition effect implies a negative slope of $R(\lambda_t)$, that is $\partial R(\lambda_t)/\partial\lambda_t < 0$.

1.5. CAPITAL MOVEMENTS AND THE COMPLETE DYNAMICAL MODEL

In a Footloose Capital model, capitalists do not move themselves, but allocate the physical capital they own between the regions. In doing so, what matters is their real net income (we assume that they take the level of the publicly provided services at home as given). Since all income is taxed and spent in the home region of the capitalist, the relevant tax rate and price index for calculating real net income are those at home, irrespective of the regional capital allocation. Therefore, in this case location choices based on real net income and on nominal gross income are equivalent.

The concrete specification of the dynamic process follows ideas from the replicator dynamics widely used in evolutionary economics and evolutionary game theory (see for example, Weibull, 1997; see also Fujita et al., 2000, p. 77, who points to this fact). Taking into account the constraint $0 \leq \lambda_{t+1} \leq 1$, the piecewise smooth one-dimensional map whereby λ_{t+1} is determined by λ_t is:

$$\lambda_{t+1} = Z(\lambda_t) = \begin{cases} 0 & if \quad F(\lambda_t) < 0 \\ F(\lambda_t) & if \quad 0 \leq F(\lambda_t) \leq 1 \\ 1 & if \quad F(\lambda_t) > 1 \end{cases} \tag{1.22}$$

where λ_t is in [0,1] implies that λ_{t+1} is in [0,1] and where

$$\frac{F(\lambda_t) - \lambda_t}{\lambda_t} = \gamma E_t = \gamma \frac{\pi_{1,t} - \left[\lambda_t \pi_{1,t} + (1-\lambda_t)\pi_{2,t}\right]}{\lambda_t \pi_{1,t} + (1-\lambda_t)\pi_{2,t}} . \tag{1.23}$$

We refer to $\gamma > 0$ as the 'speed' with which the representative capitalist alters the share of capital in region 1 in response to economic incentives E_t, in particular to a comparison of the rate of profit in region 1 with the average rate of profit, given by $\left[\lambda_t \pi_{1,t} + (1-\lambda_t)\pi_{2,t}\right]$. It can be transformed into a law of motion depending upon the ratio in regional profitability, $R(\lambda_t)$: [7]

$$F(\lambda_t) = \lambda_t + \gamma \lambda_t (1-\lambda_t) \frac{R(\lambda_t) - 1}{\lambda_t R(\lambda_t) + (1-\lambda_t)} . \tag{1.24}$$

Fixed points for the dynamic system, which correspond to points of rest or long-run equilibria, are defined by $Z(\lambda) = \lambda$. Core–periphery equilibria, that is $\lambda_0^{CP} = 0$ or $\lambda_1^{CP} = 1$, are boundary fixed points of the dynamic system. A key question of the New Economic Geography concerns critical values for trade freeness (or for any other parameter) at which agglomeration in either region is sustainable. The so-called sustain points give conditions under which 'the advantages created by such a concentration, should it somehow come into existence, [are] sufficient to maintain it' (Fujita et al., 2000, p. 9).

Sustain points therefore specify conditions at which the boundary equilibria λ_i^{CP} (where $i = 0, 1$) become (at least locally) stable. These critical values are defined by $F'\left(\lambda_i^{CP}\right)=1$, with the latter indicating the derivative of the first return map Equation (1.23). The latter condition can be reduced to $R\left(\lambda_i^{CP}\right)=1$ and solved for

$$\phi_{1,2}^{S(0)}=\frac{z\pm\sqrt{z^2-4s_E\left(1-s_E\right)}}{2\left(1-s_E\right)} \qquad \phi_{1,2}^{S(1)}=\frac{1\pm\sqrt{1-4z^2s_E\left(1-s_E\right)}}{2zs_E}, \qquad (1.25)$$

where $\phi^{S(i)}$ indicates the sustain point for λ_i^{CP}. Tables 1.1 and 1.2 and Figure 1.6 summarize the properties of the sustain values.

Table 1.1. Properties of the sustain values for the boundary fixed point $\lambda_0^{CP}=0$

Properties of $\phi_{1,2}^{S(0)}$		
$1<z$	$\phi_{1,2}^{S(0)}$ are both real and $0<\phi_2^{S(0)}<1<\phi_1^{S(0)}$ holds	
$2\sqrt{s_E\left(1-s_E\right)}<z<1$	$\phi_{1,2}^{S(0)}$ are both real	
	$s_E<0.5$: $0<\phi_2^{S(0)}<\phi_1^{S(0)}<1$	$s_E>0.5$: $1<\phi_2^{S(0)}<\phi_1^{S(0)}$
$z<2\sqrt{s_E\left(1-s_E\right)}$	No real $\phi_{1,2}^{S(0)}$ exists	

Table 1.2. Properties of the sustain values for the boundary fixed point $\lambda_1^{CP}=1$

Properties of $\phi_{1,2}^{S(1)}$		
$1<\frac{1}{2\sqrt{s_E\left(1-s_E\right)}}<z$	No real $\phi_{1,2}^{S(1)}$ exists	
$1<z<\frac{1}{2\sqrt{s_E\left(1-s_E\right)}}$	$\phi_{1,2}^{S(1)}$ are both real	
	$s_E<0.5$: $1<\phi_2^{S(1)}<\phi_1^{S(1)}$	$s_E>0.5$: $0<\phi_2^{S(1)}<\phi_1^{S(1)}<1$
$z<1$	$\phi_{1,2}^{S(1)}$ are both real and $0<\phi_2^{S(1)}<1<\phi_1^{S(1)}$ holds	

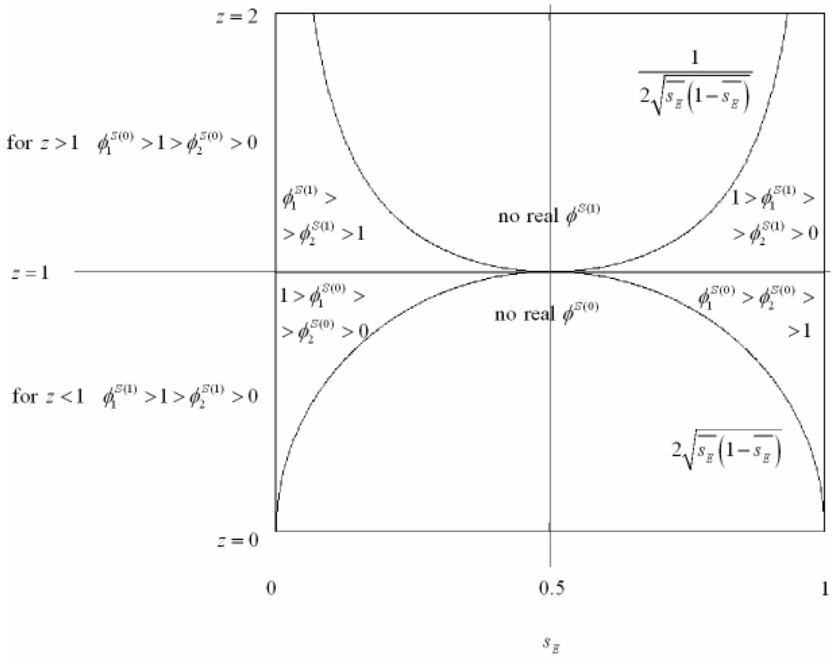


Figure 1.6. Properties of the sustain values for the boundary fixed points $\lambda_0^{CP}=0$ *and* $\lambda_1^{CP}=1$

In addition to the boundary fixed points, an interior fixed point is given by

$$\lambda^* = \frac{1}{2} + z\frac{(1-\phi)(1+\phi)}{(1-z\phi)(z-\phi)}\left[s_E - \frac{1}{2}\frac{(1+z\phi)(z-\phi)}{(1-\phi)(1+\phi)z}\right]. \tag{1.26}$$

Note that the condition $\phi<z<\phi^{-1}$ is necessary for $0<\lambda^*<1$ to hold.[8] That is, for an interior equilibrium to exist, the two regions should not differ too much in terms of provisions of public services within their territory.

A second key question of the New Economic Geography concerns critical values for the trade freeness (or for any other parameter) at which an (interior) equilibrium without spatial concentration 'breaks up'. This so-called break point gives conditions under which 'small differences among locations [will] snowball into larger differences over time, so that the symmetry between identical locations will spontaneously break' (Fujita et al., 2000, p. 9), that is it gives conditions under which an interior fixed point λ^* becomes (at least locally) unstable and the dynamics is attracted to one of the boundary equilibria. Analytically, the break point is defined by $F'(\lambda^*)=1$. In our model, the break point ϕ^B arises when the interior fixed point coincides with one of the boundary fixed points and it is equal to the corresponding sustain point. At that value of the trade freeness a transcritical bifurcation occurs (see Wiggins, 1990). Two fixed points (that is, the interior fixed point and one of the boundary fixed points) cross each other (with the interior fixed point leaving the admissible interval) and they exchange stability.

The fixed point may lose stability via a Flip bifurcation, defined by $F'(\lambda^*)=-1$; it occurs if the parameters satisfy the following condition:

$$1+\gamma\frac{\left[\phi(z-\phi)-s_E\left(1-\phi^2\right)\right]\left[z-\phi-zs_E\left(1-\phi^2\right)\right]}{zs_E\left(1-s_E\right)\left(1-\phi^2\right)^2}=-1\,. \tag{1.27}$$

Figure 1.7 presents bifurcation diagrams showing the impact of trade freeness ϕ on the long-term regional allocation of capital λ_t for (a) $z = 1$ and (b) $z = 0.98$. The other parameters are set at the values $s_E=0.4$ and $\gamma=10$ and the initial condition is close to the interior fixed point value $\lambda_0=1.005\lambda^*$.

Figure 1.7(a) is plotted for $z = 1$. Since $s_E<0.5$, from the properties of the sustain points (see Table 1.2 above),[9] it follows that:[10]

$$\phi_2^{S(0)}=\frac{s_E}{1-s_E}<\phi_1^{S(0)}=\phi_2^{S(1)}=1<\phi_1^{S(1)}$$

The boundary equilibrium $\lambda_1^{CP}=1$ is (locally) unstable for all values of ϕ. In a highly integrated economy (that is low transport costs or high trade freeness), in particular for $\phi_2^{S(0)}<\phi<1$ no interior fixed point exists within the (0, 1) interval, and the boundary equilibrium $\lambda_1^{CP}=0$ is locally stable. As ϕ crosses the sustain point $\phi_2^{S(0)}$, a transcritical bifurcation occurs: $\lambda_1^{CP}=0$ loses stability; the interior fixed point enters the (0, 1) interval and becomes locally stable. At intermediate values of trade freeness, that is, for $\phi^{Flip}<\phi<\phi_2^{S(0)}$, the interior fixed point λ^* is (locally) stable. Looking at this interval it can be noticed that, as ϕ increases, λ^* declines. The larger market size favours agglomeration in region 2.

With stronger trade barriers, as ϕ crosses ϕ^{Flip}, the interior fixed point loses stability via a Flip bifurcation. Attracting periodic solutions appear: first a two-cycle period, then – at lower values of ϕ – the time path exhibits cycles of any order and even chaotic patterns with an ever increasing volatility of the regional shares of capital. For $\phi<\phi^A$, the volatility that results for relatively high trade costs leads to the concentration of all industrial activity in one of the regions. Given the mobility hypothesis as specified in (1.22), the share of capital no longer changes once one of the boundary values 0 or 1 is reached. A core–periphery outcome emerges even though both boundary fixed points are locally unstable.[11]

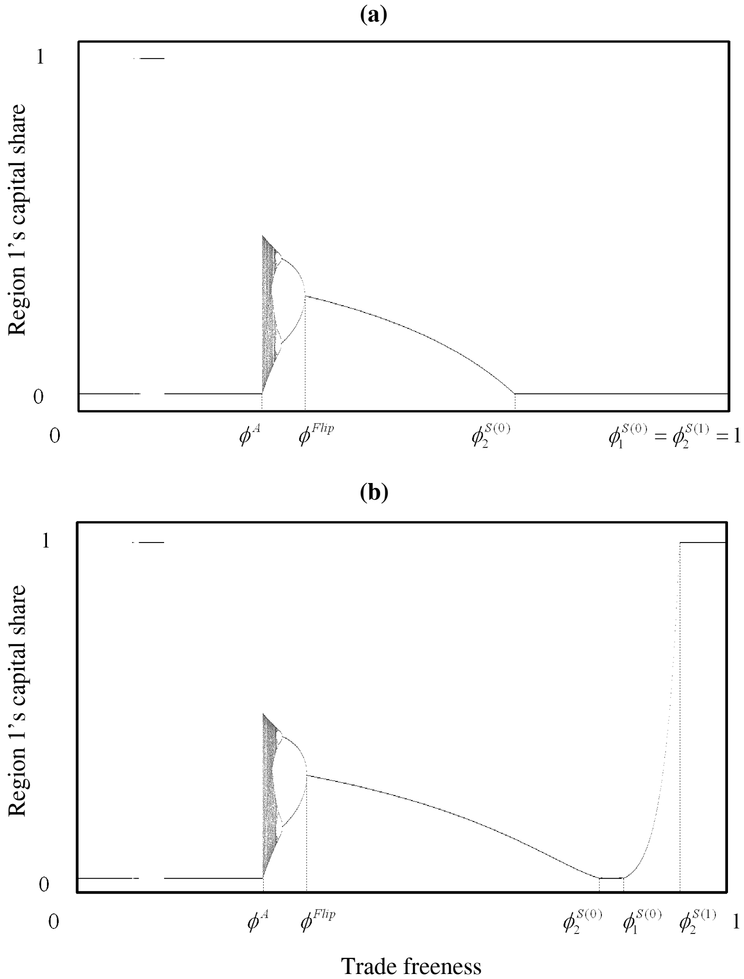


Figure 1.7. Bifurcation diagram showing the impact of trade freeness ϕ on the long-run behaviour of region 1's share of capital use λ_t for (a) z=1 and (b) z=0.98

Figure 1.7(b) is plotted for $z = 0.98$. Since $2\sqrt{s_E(1-s_E)}<z<1$, from the properties of the sustain values, it follows that:[12]

$$\frac{s_E}{1-s_E}<\phi_2^{S(0)}<\phi_1^{S(0)}<\phi_2^{S(1)}<1<\phi_2^{S(2)}.$$

For $0 \le \phi \le \phi_2^{S(0)}$ the behaviour of λ_t in Figure 1.7(b) is qualitatively similar to the behaviour in Figure 1.7(a). The most notable changes occur in the interval $\phi_2^{S(0)} < \phi < 1$. Firstly, full agglomeration in region 2 takes place at a higher value of the sustain point $\phi_2^{S(0)}$.

Secondly, even though the boundary fixed point $\lambda_0^{CP} = 0$ is stable for $\phi_2^{S(0)} < \phi < \phi_1^{S(0)}$ as in Figure 1.7(a), it loses its stability with increasing economic integration. As ϕ crosses the sustain point $\phi_1^{S(0)}$ from left to right, a transcritical bifurcation occurs: the interior fixed point re-enters the (0, 1) interval and becomes locally stable. Looking at this interval it can be noticed that, as ϕ increases, λ^* does likewise. With increasing economic integration, the productivity rise in region 1 – induced by a larger provision of public services – overcomes the disadvantage of a smaller market size and favours agglomeration in this region.

Finally, with ϕ increasing further, as ϕ crosses the sustain point $\phi_2^{S(1)}$ another transcritical bifurcation occurs. The interior fixed point exits the (0, 1) interval, losing stability, and the boundary fixed point $\lambda_0^{CP} = 1$ gains local stability. Unlike the previous case, strong economic integration causes a 'near catastrophic' agglomeration of capital from region 2 to region 1.

The differences between Figures 1.7(a) and 1.7(b), which are more considerable for $\phi_2^{S(0)} < \phi < 1$, follow from a small change in parameter z. The latter depends on H_1 and H_2 – the regional provisions of public services – which, in our model, are crucial policy tools available to the central government, together with s_F and $(1 - s_F)$, the regional contributions to the financing of public services. Parameter s_F affects region 1's expenditure share for manufactured goods and therefore the regional relative market sizes. We leave further considerations upon public policies to the next section.

1.6. THE IMPACT OF PUBLIC EXPENDITURE ON INDUSTRIAL LOCATION

In order to understand the impact of public expenditure on the concentration of industrial activity in our model, we first consider the interior equilibrium λ^*. The provision of public services in region 1 affects λ^* as follows:

$$\frac{\partial \lambda^*}{\partial H_1} = -\frac{\phi}{(1-\phi z)^2}\left[1 - \frac{(z^2-1)(1-\phi^2)}{(z-\phi)^2} s_E\right]\frac{\partial z}{\partial H_1} + z\frac{1-\phi^2}{(1-\phi z)(z-\phi)}\frac{\partial s_E}{\partial H_1} \qquad (1.28)$$

Since it is the sum of two addends, it is possible to identify neatly two effects that an increase in H_1 could exert on the location of the manufacturing sector. According to the 'productivity effect', expressed by the first addend in Equation (1.28), the provision of public services in region 1 affects λ^* via its effect on the labour productivity in the manufacturing sector located in region 1. Since the term in the square brackets is positive for any $0 \leq s_E \leq 1$ and $(\partial z / \partial H_1) < 0$, the productivity effect is positive on λ^*.

According to the 'demand effect', expressed by the second addend in Equation (1.28), the provision of public services in region 1 affects λ^* via a change in the relative market size. Since $\phi < z < \phi^{-1}$ and $\phi < 1$, the sign of the demand effect corresponds to the sign of $\partial s_E / \partial H_1 = (s_L - s_F)[(\sigma - \mu)/\sigma][L/(L-H)^2]$. Therefore, for $s_F > s_L$, the demand effect is negative on λ^*. Conversely, for $s_F < s_L$, the demand effect is positive on λ^*. For $s_F = s_L$ no demand effect occurs.

In order to disentangle the relative importance of these two effects, in the following analysis we employ numerical simulations and analyse the following stylized case: region 1 is the 'poor' region, in the sense of having the lower income share without any public expenditure. In order to improve the situation in region 1, public expenditure that enhances productivity in region 1 is increased. We study whether and how the effect of such a policy depends upon its financing scheme. We set $s_L = 0.5$, $s_K = 0.25$, $\sigma = 4$ and $\mu = 0.5$, hence $\widetilde{s_E} = 0.46875$. That is, since $\widetilde{s_E} < 0.5$, region 1 is the poor region since it has a smaller factor endowment than region 2. We also set $\phi = 0.2$, $L = 1$, $A = 1$, $B = 0.45$ and $H_2 = 0$, that is region 2 receives no public expenditure.

Figure 1.8 summarizes the effects on λ^* of an increase in the provision of public services in region 1, within the interval $0 \leq H_1 < L$,[13] for different values of the regional tax burden necessary to finance it, s_F. The solid line corresponds to $s_F = s_L = 0.5$, that is, the burden of taxation is equally distributed among consumers in the two regions. The demand effect is nil and only the productivity effect impact (positively) on region 1's share of capital. The dotted line corresponds to $s_F = 0.55$. The demand effect is negative and after an initial range it overcomes the productivity effect. Finally, the dashed line corresponds to $s_F = 1$, and the demand effect is stronger than the productivity effect for any H_1 in the interval considered.[14]

The latter result tells us that policy measures aimed at enhancing labour productivity of manufacturing firms in the backward region will be effective only if the prosperous region participates in the financing of such policies. If the poor region is 'left alone' (that is, if public services are financed solely by the income of residents in that region – corresponding to $s_F = 1$ in our simulation study), then the demand effect of an increase in H_1 will eventually prevail. On the other hand, if the government sets s_F at a sufficiently small value, making region 2 taxpayers contribute according to their capacity (s_F equal to s_L or smaller – in our simulation study we

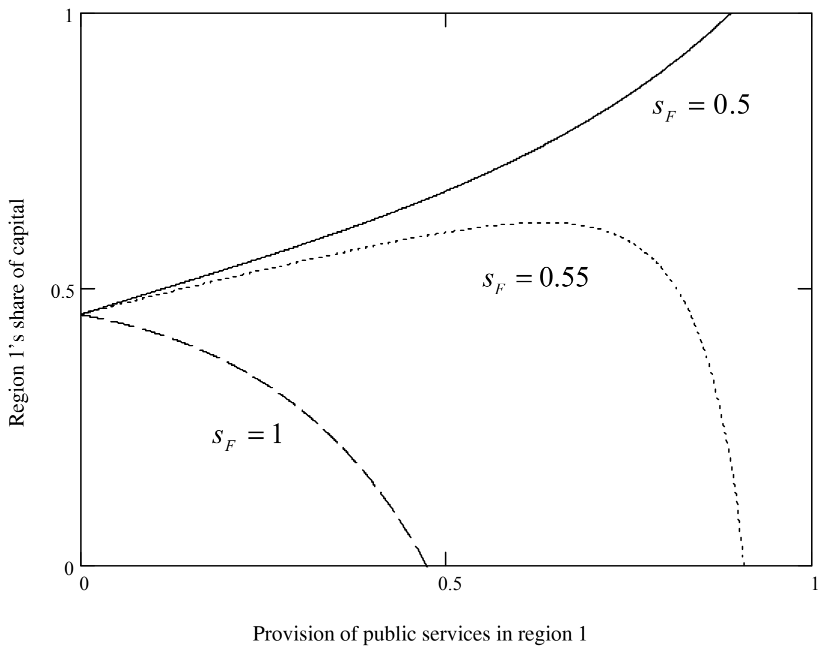


Figure 1.8. Diagram showing the impact of the provision of public services in region 1 on the interior equilibrium λ^* for different values of region 1's share of the tax burden necessary to finance public services

explored the case $s_F = s_L = 0.5$), then the demand effect of public expenditure is negligible or it could even act in the same direction as the productivity effect.

We now consider the impact of the provision of public services on the long-term behaviour of region's 1 share of capital λ_t. Figure 1.9 confirms the above conclusions. That is, increasing H_1 could favour agglomeration in region 1 as long as the increase is not too large. Otherwise the demand effect could overcome the productivity effect, inducing agglomeration of the manufacturing sector in region 2. As shown in Figure 1.9(a), plotted for ϕ=0.3, for $0<H_1<H_{1,2}^{S(1)}$ increasing the provision of public services favours the location of the industrial activity in region 1.[15] For $H_1^{Flip}<H_1<H_{1,2}^{S(1)}$, the interior fixed point is locally stable, whereas for $0<H_1<H_1^{Flip}$ complex behaviour takes place in the long run.[16] As H_1 crosses $H_{1,2}^{S(1)}$ a transcritical bifurcation takes place: the interior fixed point leaves the (0, 1) interval, losing stability; and the boundary fixed point $\lambda_0^{CP}=1$ becomes locally stable.

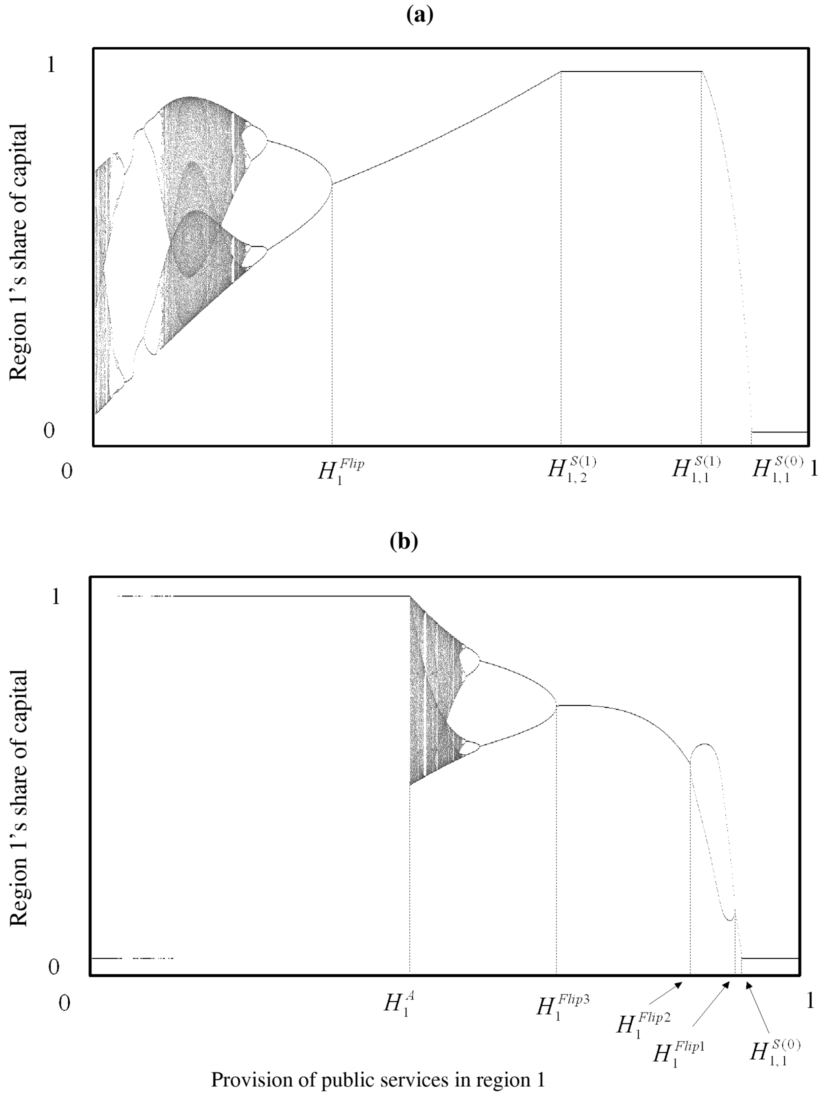


Figure 1.9. Bifurcation diagram showing the impact of the provision of public services in region 1 H_1 *on the long-run behaviour of region 1's share of capital use* λ_t *for (a)* ϕ=0.3 *and (b)* ϕ=0.23

Compared with the behaviour of λ^* in Figure 1.8, for which $\phi = 0.2$, a higher value of ϕ causes full agglomeration of the industrial activity in region 1 by strengthening the productivity effect.

However, increasing H_1 further reinforces the demand effect, eventually offsetting the productivity effect. As H_1 crosses $H_{1,1}^{S(1)}$, the boundary fixed point, $\lambda_0^{CP}=1$ loses stability; the interior fixed point re-enters the (0, 1) interval and gains local stability. The increase in taxation necessary to finance the provision of public services reduces substantially region 1's market size. Therefore while H_1 rises, the manufacturing sector shifts progressively to region 2. Finally, as H_1 crosses the sustain point $H_{1,1}^{S(0)}$ another transcritical bifurcation takes place. The interior fixed point exits the (0, 1) interval, losing stability; and the boundary fixed point $\lambda_0^{CP}=0$ gains stability. All manufacturing migrates to region 2 through a 'near catastrophic' process of agglomeration.

Figure 1.9(b), which is plotted for a smaller value of trade freeness (that is, $\phi=0.23$), differs from Figure 1.9(a) in two important respects. First, the productivity effect is weaker and the demand effect stronger. Therefore, when the provision of public services is relatively large, full agglomeration in region 1 never occurs. Moreover, full agglomeration in region 2 takes place through a smoother process. Second, the long-term behaviour of the capital shares is considerably more volatile. The first Flip bifurcation value H_1^{Flip3} is much higher. Moreover, within the interval $H_1^{Flip3}<H_1<H_{1,1}^{S(0)}$, two other Flip bifurcations occur for the interior fixed point. Finally, for $0\leq H_1<H_1^A$ the volatility of region 1's capital share is so high that, sooner or later, it converges on either $\lambda_0^{CP}=0$ or $\lambda_1^{CP}=1$.[17]

1.7. CONCLUSIONS

This chapter delivers further insights into the impact of productive public spending on industrial location, extending the results provided by Commendatore et al. (2008) in two main directions: it confirms their conclusions within a much easier framework and examines the dynamic behaviour of relevant economic variables.

We dealt with an FC model without investment goods and with a government using the agricultural good as an input in the provision of public services. Our main contributions were the following.

First, we fully characterized the dynamics of capital movements in the model, looking at stability properties of industrial location equilibria, under alternative degrees of economic integration.

Second, we decomposed the overall effect of an increase in the endowment of productivity enhancing public services available to firms located in backward regions into two components: the productivity effect and the demand effect. In other words, firms are attracted by lower input requirements, while higher taxation tends to shrink the local market, leading firms to relocate elsewhere. As in Commendatore et al. (2008), the demand effect will be nil only if the tax burden is equally distributed across regions. On the other hand, the demand effect of public expenditure will be offset by (or even act in the same direction as) the productivity effect if the government makes the taxpayers in the richer region contribute according to their capacity.

Finally, the above policy analysis was extended to a dynamic context, studying the long-term regional relocation of capital induced by public spending, allowing the degree of economic integration to vary. In doing so, we sought to draw a full picture of the relative strength of the productivity and the demand effects under alternative scenarios of trade freeness.

NOTES

1. As the authors point out, while billions of euros are devoted to financing infrastructure projects in core regions, the bulk of the available funds is directed towards lagging regions in an attempt to achieve a 'balanced spread of economic activity' (Brakman et al., 2005).
2. See, in particular, the contributions of Martin and Rogers (1995), Trionfetti (1997), Brülhart and Trionfetti (2004), Brakman et al. (2007) and Commendatore et al. (2008). For an overview on the main results in the literature the reader is referred to Commendatore et al. (2008).
3. The direction of the demand effect could even be reversed, moving in the same direction as the productivity effect and favouring agglomeration in the backward region, if taxpayers in the richer country contribute in a larger proportion than their contributive capacity.
4. According to a recent document of the European Commission, '[c]ohesion policy is aimed at supporting three main areas of investment: infrastructure (mainly transport and the environment), productive investment (largely SMEs and RTD and innovation) and investment in people. Over the period 2000–2006, investment was concentrated in these three areas in both Objectives 1 and 2, though with differing emphasis. Whereas in Objective 1 regions, the focus was on basic needs, on infrastructure (particularly transport infrastructure) and human resources, in Objective 2 regions, investment was centered more on 'soft' infrastructure, particularly on aid to SMEs and RTD. Objective 3 was dedicated in turn to human resources' (European Commission, 2007, p. 93)
5. These figures refer to 'gross fixed capital formation' (dwellings, other buildings and structures, machinery and equipment, and computer software).
6. As a result of Walras' Law, equilibrium in all product markets implies equilibrium in the regional labour markets.
7. Note that – from an analytic perspective – this specification is a good approximation to the discrete-time counterpart of the process assumed by Puga (1998) in his core–periphery model.
8. This can be easily verified given that $0<\lambda^*<1$ for

$$\phi\frac{z-\phi}{1-\phi^2}<s_E<\frac{1}{z}\frac{z-\phi}{1-\phi^2}.$$

Indeed, a necessary condition for these inequalities to hold is $\phi<z<\phi^{-1}$.

9. Since they coincide with the sustain points, we do not mark the break points in the figures.
10. From $z = 1$ and $s_E<0.5$, it follows that

$$\lambda^* = \frac{1}{2}+\frac{1+\phi}{1-\phi}\left(s_E-\frac{1}{2}\right)$$

and

$$\frac{\partial\lambda^*}{\partial\phi}=-\frac{1-2s_E}{(1-\phi)^2}<0$$

Given that $\lambda^*=s_E$ at $\phi=0$ and $\lambda^*\to-\infty$ for $\phi\to1$, the interior equilibrium curve λ^* in Figure 1.7(a) necessarily cuts the 0 line when $0<\phi<1$.

11. A more detailed account of the phenomenon of agglomeration via volatility for the standard Footloose Capital model with no government sector is presented in Commendatore et al. (2007).
12. For $z<1$ and $s_E<0.5$, the interior equilibrium curve in Figure 1.7(b) has a relative minimum at $\phi_{\min}$ and a relative maximum at $\phi_{\max}$, where $0<\phi_{\min}<z<\phi_{\max}<1$ and where

$$\phi_i=\frac{z(1-2s_E)\pm(1-z)\sqrt{s_E(1-s_E)}}{1-(1+z^2)s_E}\quad \text{and } i = \text{max, min}$$

When $z>2\sqrt{s_E(1-s_E)}$, the interior equilibrium curve – which at $\phi=0$ starts from $\lambda^*=s_E$ – cuts the 0 line from above. This is because its (relative) minimum is negative, $\lambda^*<0$ at $\phi_{\min}$. The curve then cuts the 0 line from below and after the 1 line from below before leaving the interval (0, 1). This is because $\lambda^*\to\infty$ for $\phi\to z$.

13. H_1 should be smaller than L in order to have positive world expenditures for manufactures, $M > 0$.
14. However, in simulations not presented here we have found that, by increasing ϕ, there could be an initial range of values of H_1 for which λ^* increases even for $s_E=1$. In this case, it could be possible to identify a positive level of H_1 that maximizes λ^*.
15. For H_1 there can be a maximum of four sustain points (and break points): two for the boundary equilibrium $\lambda_0^{CP}=0$ and two for $\lambda_1^{CP}=1$. In Figure 1.9(a) three of them are visible, $H_{1,1}^{S(0)}$, $H_{1,1}^{S(1)}$ and $H_{1,2}^{S(1)}$; whereas in Figure 1.9(b) only one, $H_{1,1}^{S(0)}$.
16. There are four values of H_1 which satisfy condition (1.27). In Figure 1.9(a) only one of them is visible, H_1^{Flip}; whereas in Figure 1.9(b) three of them are: H_1^{Flip1}, H_1^{Flip2} and H_1^{Flip3}.
17. In this interval, the long-term location of the overall manufacturing sector is highly sensitive to the initial condition λ_0. See Commendatore et al. (2007).

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