6. Government spending, effective demand, distribution and growth: a dynamic analysis

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6.1. INTRODUCTION

This chapter deals with the role of different types of government expenditure in post-Keynesian analysis. This tradition of thought has proposed a formal treatment of these issues with significant delay, in spite of the crucial importance it attributes to government intervention and the fact that its founders paid great attention to the problem of the composition of government spending.[1]

Steedman (1972) made the first attempt to provide a formal treatment of the intervention of a government sector with a balanced budget in the post-Keynesian theory of growth and distribution. Some years later, Fleck and Domenghino (1987) and Pasinetti (1989) started an intense debate on the formal analysis of government deficit within this theory (see Panico, 1997) and Commendatore et al., 2003), while You and Dutt (1996), Lavoie (2000) and Commendatore et al. (2005) proposed a formal analysis of how government debt can affect income distribution and the rate of growth within alternative post-Keynesian approaches. Very recently some post-Keynesian analyses of growth and distribution have focussed on the role of government expenditure and on complex forms of dynamics, generating hysteresis, poverty traps and so on (see Skott, 2008; Hein, Lavoie and Treek, 2008; Commendatore, 2006; Commendatore et al., 2007 and 2009). These contributions have attained interesting results in the analysis of equilibrium conditions and have paved the way for dynamic treatment of these problems.

Commendatore et al. (2007; 2009) argue that it is important to distinguish different kinds of government expenditures, showing that in the Kaleckian analysis increases in what has been termed ‘unproductive’[2] expenditure have a positive influence on growth and can generate hysteresis effects which permanently lead the economy out of the poverty trap. Increases in 'productive' expenditure produce the same positive effects, but may also reduce the degree of capital utilisation and the rate of growth depending on the distribution of the productivity gains between wages and profits. In the Classical–Harrodian analysis increases in 'unproductive' expenditure do not produce hysteresis effects and there can be both positive and negative influences on the degree of capital utilisation and the rate of growth. The same effects are produced by increases in 'productive' expenditure, with the difference that in this case their occurrence depends on whether after-tax profits rise or diminish.

In what follows we attempt a dynamic analysis of the role played by different kinds of government expenditure in these post-Keynesian theories. It is argued that in Kaleckian analysis increases in 'unproductive' expenditure have a positive influence on the stability of the system, such that the existence of a 'big government' favours both the growth and the controllability of the economy.[3] The influence of increases in 'productive' expenditure on the stability and controllability of the system are not so clear-cut, but depend on whether after-tax profits rise or fall. In the Classical–Harrodian analysis increases in 'unproductive' expenditure improve the controllability of the economic system when the size of the government sector is sufficiently large. In this case, however, the effects on growth can be both positive and negative. Examination of changes in 'productive' expenditure shows instead that increases positively affect the stability of the economy when the size of the government sector is sufficiently large, whereas reductions have the same effects when the government sector is sufficiently small. Moreover the controllability of the system improves when the size of the government sector approaches the level that maximises after-tax profits.

The chapter is organised as follows. Section 6.2 presents the analytical model. Sections 6.3 and 6.4 elaborate the Kaleckian and Classical–Harrodian analysis respectively. Section 6.5 summarises the main conclusions.

6.2. THE MODEL

The model used by Commendatore et al. (2007; 2009) refers to a single-good closed economy with two production inputs: labour, whose supply is perfectly elastic, and fixed capital, which does not depreciate. In this model technical progress is excluded, in each period the capacity of the capital stock is not fully utilised, such that the potential and current output do not coincide, and the production function is of a Leontief type:

$$Y^P = aK \quad and \quad Y = bL \tag{6.1}$$

where Y^P is the potential output, Y the current output, K the stock of capital, L the amount of labour employed in production, and a and b are the reciprocal of the capital and labour coefficients respectively.

The degree of capacity utilisation is defined as:

$$u = \frac{Y}{Y^P} \tag{6.2}$$

Income is distributed between wages and profits: $Y = wL + rK$, where w is the wage rate and r is the rate of profit. Normalising with respect to output, and taking into account expressions (6.1) and (6.2), this equation becomes

$$1 = \frac{w}{b} + \pi \tag{6.3}$$

where w/b and $\pi \equiv r/au$ are respectively the share of wages and the share of profits in national income.

The model assumes that the wage rate is a function of labour productivity:

$$w = w(b) \quad with \quad w'(b) \geq 0$$

where the value of $w'(b)$ depends on the bargaining power of the unions. By letting the wage-productivity elasticity $\lambda \geq 0$ measure the ability of unions to capture labour productivity improvements, we write

$$w = w_0 b^{\lambda} \tag{6.4}$$

When $0 \leq \lambda < 1$, workers are not able to fully capture the increase in productivity; when instead $\lambda \geq 1$, wage increases are equal or higher than productivity improvements.

The model further assumes that the government sector operates under a balanced budget constraint:

$$\tau = \gamma \tag{6.5}$$

where taxation τ and public expenditure γ are expressed in terms of income.

Although government expenditure can affect labour and capital productivity, the model focuses on the influence on average labour productivity b:

$$b = b(\gamma) \quad \text{with} \quad b(0) > 0, \quad b' \geq 0 \quad \text{and} \quad b'' \leq 0 \tag{6.6}$$

When $b' = 0$, government expenditure has no effect on the labour input coefficient; when $b' > 0$, government expenditure reduces the labour coefficient and increases the productivity of this input.

Like in Barro (1990, p. S107), the model assumes that the government enhances input productivity by purchasing goods and services that are freely provided to the private sector. It describes these features by introducing a production function where government expenditure may reduce, as a positive externality, the labour and capital coefficients, instead of following Barro's choice to allow government expenditure to explicitly enter the production function as an argument.

When government expenditure affects labour productivity, the wage share may vary too, depending on the bargaining power of the unions, i.e. on λ. This assumption makes it possible, using equations (6.3), (6.4) and (6.6), to describe the profit share as a function of government expenditure:

$$\pi = \pi(\gamma) = 1 - w_0 \left[b(\gamma) \right]^{\lambda - 1} \tag{6.7}$$

where

$$\pi'(\gamma) = (1-\lambda) w \frac{b'(\gamma)}{b} \geq (<) 0 \quad for \quad \lambda \leq (>) 1$$

If the wage rate increases less (more) than labour productivity, the profit share increases (decreases). In the analysis of the equilibrium condition developed by Commendatore et al. (2007; 2009) and in that proposed below, this relationship plays a crucial role.

As to the private sector, the model assumes that workers do not save and the investment function is not linear:[4]

$$s = s_\pi r (1-\tau) = s_\pi \pi a u (1-\tau) \tag{6.8}$$

$$g = \alpha + \phi(u) \tag{6.9}$$

where s is the saving to capital ratio and s_π is the propensity to save out of profits while, according to equation (6.9), capital accumulation g depends on an autonomous term α and on a non-linear term $\phi(u)$ enjoying the following properties:

$$\phi(\tilde{u}) = 0, \;\; \phi' > 0, \;\; \text{and} \;\; \phi'' \geq (<) 0 \;\; \text{for} \;\; u \leq (>) \tilde{u}$$

where $0 < \tilde{u} \leq 1$ is the *normal* degree of capacity utilisation that is interpreted as the optimal degree of capacity utilisation given the existing technology (for further details, see Commendatore et al., 2007; 2009).

The term α can be interpreted along a Kaleckian or a Classical–Harrodian line. In the first case, α reflects entrepreneurs' animal spirits and is taken as given like the state of long-term expectations in Keynesian models (see Rowthorn, 1981; Dutt, 1984; Amadeo, 1986; and Lavoie, 1992):

$$\alpha = \bar{\alpha} \tag{6.10}$$

In the second case, α represents the Harrodian 'warranted rate of growth', which depends on the saving generated at normal capacity utilisation (for a similar interpretation, see Commendatore et al., 2003, Shaikh, 2007):

$$\alpha = \tilde{g} \equiv s_\pi \tilde{r}(1-\tau) \tag{6.11}$$

where $\tilde{g}$ is the warranted rate of growth and $\tilde{r} \equiv \pi a \tilde{u}$ is the rate of profit corresponding to normal capacity utilisation.

The dynamics of the system is generated by the variations in the degree of capital utilisation in the face of discrepancies between demand and supply, i.e., between investment and saving. If in one period the economy is not in equilibrium, in the following period the degree of capacity utilisation changes:

$$u_{+1} = \psi(u) = u + \theta(g - s) \tag{6.12}$$

where 'x_{+1}' denotes the one-period forwarded value of the variable x and where $\theta > 0$ is the speed at which capacity utilisation adjusts to the discrepancy between saving and investment.

By imposing in each period the equilibrium condition $g = s$ and given that $r \equiv \pi a u$, Commendatore et al. (2007; 2009) show that the solutions for u and g correspond to:[5]

$$u^* = \frac{\alpha + \phi(u^*)}{s_\pi \pi a(1-\gamma)} \qquad g^* = s_\pi \pi a(1-\gamma)u^* \tag{6.13}$$

where 'x^*' denotes the equilibrium value of the variable x. The existence of equilibrium solutions, whether single or multiple, depends on the value of the parameters.

An equilibrium solution is locally asymptotically stable or attracting if, and only if, it satisfies the following condition:

$$0 < \theta\left[s_\pi \pi a(1-\gamma) - \phi'(u^*)\right] < 2 \tag{6.14}$$

Expression (6.14) implies that a necessary condition for local stability is that at the equilibrium the slope of the saving function should be steeper than that of the investment function:

$$s_\pi \pi a(1-\gamma) > \phi'(u^*) \tag{6.15}$$

6.3. THE KALECKIAN INTERPRETATION

6.3.1. Equilibrium

In the Kaleckian interpretation α reflects entrepreneurs' animal spirits and represents the expected growth rate of demand. In this case the equilibrium solutions are

$$u^* = \frac{\overline{\alpha} + \phi(u^*)}{s_\pi \pi a(1-\gamma)} \qquad g^* = s_\pi \pi a(1-\gamma)u^* \tag{6.16}$$

Commendatore et al. (2007; 2009) present the diagram reported in Figure 6.1, which shows that, according to the value attributed to γ, there can up to three equilibrium positions that the economy can reach, denoted by $e_L \equiv (u_L^*, g_L^*)$, $e_I \equiv (u_I^*, g_I^*)$ and $e_H \equiv (u_H^*, g_H^*)$.[6] The intermediate equilibrium e_I is unstable, because condition (6.15) is violated. For the 'low' equilibrium e_L and the 'high' equilibrium e_H, the slope of the saving function is steeper than that of the investment function. These equilibrium positions are locally stable if conditions (6.14) and (6.15) are satisfied.

Commendatore and al. (2007; 2009) also show that by changing the value of the parameters one or two equilibrium positions may disappear via a fold bifurcation. The bifurcation sequence depends on the impact of government expenditure on the labour coefficient. In Figure 6.1(a), when γ does not affect labour productivity, there is one equilibrium position e_L. The rise of γ generates a fold bifurcation and the appearance of the intermediate and high equilibrium positions, e_I and e_H (Figure 6.1b). Further increases in γ end up

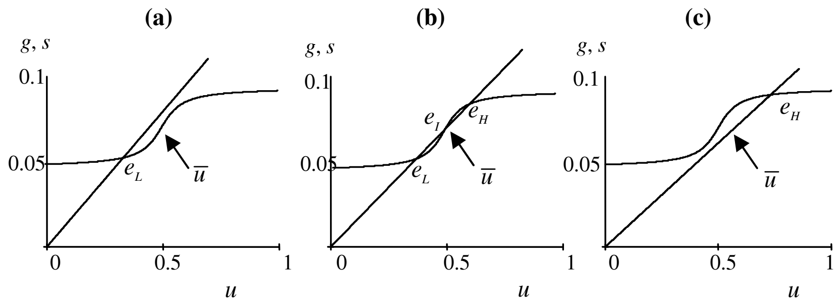

Figure 6.1. Kaleckian interpretation: saving and investment functions for different values of public expenditure. (a) only the low equilibrium exists; (b) three equilibria are present (low, intermediate, high); (c) only the high equilibrium exists

generating another fold bifurcation and the disappearance of the low and intermediate equilibrium positions (Figure 6.1c). Notice, however, that changes in γ that affect labour productivity can also generate the opposite tendency, going from one high equilibrium position to one low equilibrium when γ increases.

These findings (Commendatore et al., 2007; 2009) show that there can be hysteresis effects, which can lead the economy into a poverty trap or out of it, depending on the influence of government expenditure on labour productivity.

6.3.2. Dynamic Analysis

Let us now present a dynamic analysis, first investigating how changes in parameter θ, representing the speed at which firms adjust capacity utilisation, affect the local stability of the equilibrium solutions and the long-term behaviour of the economy. The analysis then considers how changes in government expenditure, γ, affects the local stability of the equilibrium positions.

The difference equation (6.12) is revised as follows for the Kaleckian case:

$$\psi(u) = u + \theta\left[\alpha + \phi(u) - s_\pi \pi a(1-\gamma)u\right] \tag{6.17}$$

This map determines the long-term evolution of the state variable u and of the economic system. θ does not affect the fixed point solution $\psi(u^*) = u^*$, which is determined by the equilibrium condition $g = s$, but it affects the local stability properties of the fixed points and the global stability properties of the map $\psi(u)$. For the fixed point u^*, a necessary and sufficient condition for local stability is

$$0 < \theta < \theta^F \equiv \frac{2}{s_\pi \pi a(1-\gamma) - \phi'(u^*)}$$

As long as this inequality is satisfied, the fixed point u^* is attracting. When $\theta > \theta^F$, a so-called flip or period doubling bifurcation occurs: u^* loses stability and a locally stable 2-cycle emerges. This means that the economic system oscillates between two states, i.e. two levels of capacity utilisation. By further increasing θ, many period-doubling bifurcations occur: the periodicity of the cycle, i.e. the number of states among which the economic system oscillates before going back to the initial one, doubles. Finally, for sufficiently high values of θ, cycles of any periodicity and even chaos emerge.[7]

Figure 6.2 considers what happens when the values of γ generate one equilibrium position and one attractor, as described in Figures 6.1(a) and 6.1(c). It presents two bifurcation diagrams, which describe the impact of the adjustment speed θ – changing within the interval $14.5 \le \theta \le 23.5$ – on the long-term behaviour of the state variable u.[8] A period-doubling route to chaotic behaviour can occur both for the low equilibrium u_L^* (Figure 6.2(a)) and for the high equilibrium u_H^* (Figure 6.2(b)).

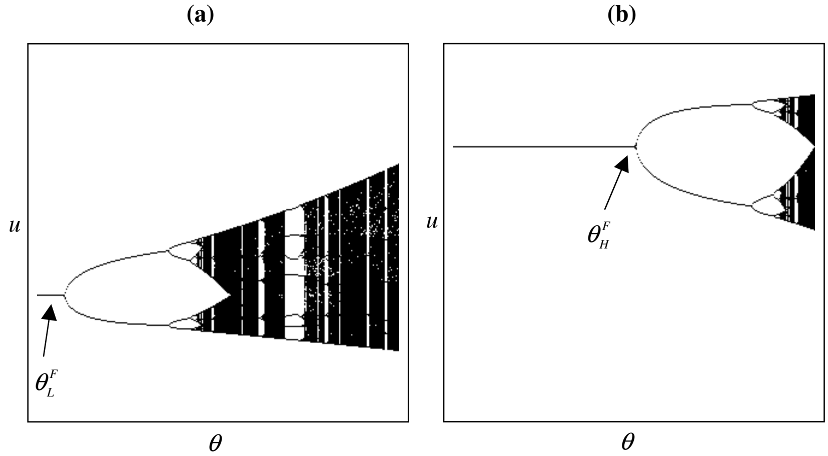

Figure 6.2. Kaleckian interpretation: bifurcation diagrams showing the impact of the adjustment speed θ on the long-term behavior of the state variable u for different values of public expenditure γ. (a) only the low equilibrium exists; (b) only the high equilibrium exists

θ_L^F and θ_H^F denote the flip bifurcation values of θ corresponding to the low and high equilibrium.

Instead, for values of γ producing three equilibrium positions, as described in Figure 6.1(b), the following situation occurs: when θ is below the smaller of θ_L^F and θ_H^F, the low and high equilibrium are both stable; for values of θ between θ_L^F and θ_H^F a stable fixed point and an attracting cycle coexist; finally, when θ is above the larger of θ_L^F and θ_H^F, two attracting cycles that can be regular or chaotic coexist. The two attractors are asymmetric with respect to the intermediate equilibrium and do not generally have the same periodicity. Moreover, the basins of the two attractors, defined as the set of the initial values of the state variable u that converge to an attractor, undergo substantial modifications as θ changes. For small values of θ, the two basins are simple and correspond to the interval to the left and the interval to the right of the intermediate equilibrium u_I^*. As θ increases, they become increasingly disconnected and intermingled.[9]

Let us now explore the influence of a change in γ on the dynamic properties of the map $\psi(u)$. Consider first the case in which government expenditure does not affect the labour input coefficient. For both θ_L^F and θ_H^F the following derivative holds:

$$\frac{d\theta^F}{d\gamma} = \theta^F \frac{s_\pi \pi a + \phi''(u^*) du^* / d\gamma}{s_\pi \pi a(1-\gamma) - \phi'(u^*)}.$$

Since condition (6.15) is satisfied and $du^* / d\gamma > 0$ for both the low and the high equilibrium, the sign of the derivative is positive for the low equilibrium where $\phi''(u_L^*) > 0$ and is indeterminate for the high equilibrium because $\phi''(u_H^*) < 0$. This means that when the low equilibrium position prevails, a rise in government expenditure increases the values of θ at which bifurcations take place. Instead, when the high equilibrium prevails, the effects of a rise in government expenditure are ambiguous.

Some simulations further clarify how a change in γ influences the dynamic properties of the map $\psi(u)$ and what the implications are of the presence of a 'big government' for the long-term behaviour of the economic system. Figure 6.3, which deals with the case in which government expenditure does not affect the production coefficients, presents three bifurcation diagrams describing the impact of γ on the long-term behaviour of the degree of capacity utilisation for $0 \leq \gamma \leq 0.4$ and for (a) $\theta = 10$, (b) $\theta = 15$ and (c) $\theta = 20$.[10] Summarising the results achieved in Commendatore et al. (2007; 2009), figure 6.3(a) shows that as γ overtakes γ^T, a fold catastrophe takes place. The economic system rapidly moves from a low to a high equilibrium position and stays there even if γ returns to its initial value. This phenomenon is known as *hysteresis*.

Figure 6.3(b) points out that at γ_L^F a flip bifurcation point takes place. For $0 \leq \gamma < \gamma_L^F$ a locally stable 2-cycle occurs. For $0 \leq \gamma < \gamma_L^F$ the 2-cycle disappears and the economic system converges towards a unique equilibrium. Analysis of $0 \leq \gamma < \gamma^T$ is also described by Figure 6.3(c) for $\theta = 20$, which confirms that the low equilibrium position is locally unstable and shows that the economic system has a chaotic behaviour. Nonetheless, increases in γ reduce the size of fluctuations, contributing to rid the economic system of chaotic behaviour. Figure 6.3(c) also shows that for $\gamma > \gamma^T$ two bifurcations occur. For $\gamma > \gamma^{F1}$ the economic system moves from a unique equilibrium position to a 2-cycle situation. For $\gamma > \gamma^{F2}$ the economic system returns to a unique equilibrium.

Figure 6.3(b) and 6.3(c) confirm that increases of γ tend to produce a positive phenomenon of hysteresis. Moreover, they tend to reduce the

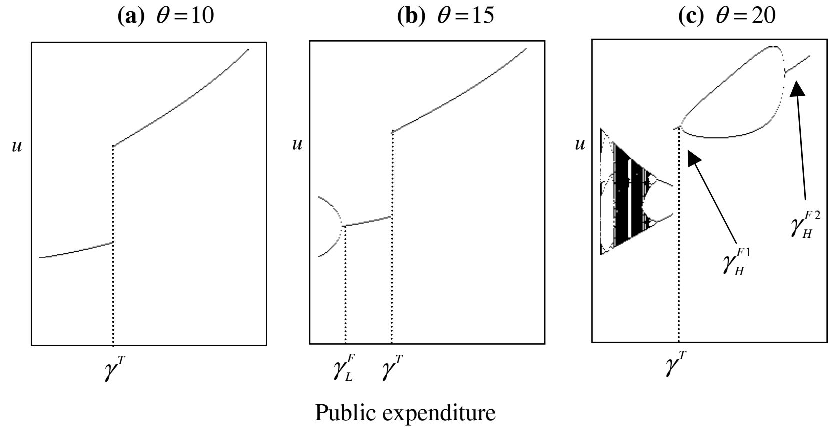

Figure 6.3. Kaleckian interpretation: bifurcation diagrams showing the impact of 'unproductive' public expenditure γ on the long-term behaviour of the state variable u for different values of the adjustment speed θ

complexity of the behaviour of the economic system, thus making it more controllable, and eventually lead the economic system to a unique and stable equilibrium.

To sum up, the previous analysis of the effects of changes of what has been called 'unproductive' government expenditure, i.e. that generating $b'=0$, attributes to a 'big government' a positive influence on the growth, stability and controllability of the economic system. The influence of changes to what has been called 'productive' government expenditure, i.e. that generating $b'(\gamma)>0$, depends instead on the effects of taxation on the disposable income of the private sector and on how the gains in productivity are distributed between profits and wages. These effects are captured by the following derivative, which holds for the flip bifurcation values of θ corresponding to the low and the high equilibrium positions:

$$\frac{d\theta^F}{d\gamma}=\theta^F\frac{s_\pi a\left[\pi(\gamma)-\pi'(\gamma)(1-\gamma)\right]+\phi''(u^*)\dfrac{du^*}{d\gamma}}{s_\pi\pi(\gamma)a(1-\gamma)-\phi'(u^*)} \tag{6.18}$$

The expression contained in the square brackets represents the impact of government expenditure on after-tax profits.

Commendatore et al. (2007; 2009) show that the relationship between after-tax profits and government expenditure is increasing for $0\le\gamma<\overline{\gamma}$ and decreasing for $\gamma>\overline{\gamma}$, where $\overline{\gamma}$ defines the value of public expenditure for which after-tax profits are maximised.

In derivative (6.18) for $0 \leq \gamma < \bar{\gamma}$ after-tax profits are positively affected and the degree of capacity utilisation is negatively affected by changes in government expenditure, hence $d\theta_L^F / d\gamma < 0$ and $d\theta_H^F / d\gamma$ is indeterminate. On the contrary, if after-tax profits are negatively affected and the degree of capacity utilisation is positively affected by changes in government expenditure, $d\theta_L^F / d\gamma > 0$ and $d\theta_H^F / d\gamma$ is indeterminate. This means that when $\gamma < \bar{\gamma}$, increases in government expenditure have a destabilising effect on the low equilibrium since they reduce the value of θ at which a flip bifurcation takes place. When instead $\gamma > \bar{\gamma}$, increases in government expenditure have a stabilising effect on the low equilibrium. The effects are always indeterminate when the high equilibrium position prevails.

Some simulations further clarify the effects of changes in government expenditure on the long-term behaviour of the degree of capacity utilisation and of the economic system. They are described in Figure 6.4, where $0 \leq \gamma \leq 0.4$ and (a) $\theta = 15$, (b) $\theta = 17.5$ and (c) $\theta = 20$. [11]

Figure 6.4(a) summarises the findings of Commendatore et al. (2007; 2009). It shows that two fold catastrophes occur for values of $\gamma = \gamma^{T1}$ and $\gamma = \gamma^{T2}$. For $\gamma \geq \gamma^{T1}$ the economic system rapidly moves from a high to a low equilibrium position and remains there even if γ returns to its initial value. The opposite effect takes place for $\gamma \geq \gamma^{T2}$, where the economic system rapidly moves towards a high equilibrium position.

Figure 6.4(b) points out that two bifurcations occur for the low equilibrium positions. For γ_L^{F1} the equilibrium loses stability and an attracting 2-cycle emerges. For γ_L^{F2} the cycle disappears and the equilibrium becomes attracting again. The size of fluctuations increases up to $\bar{\gamma}$ and decreases afterwards.

Analysis of $\gamma^{T1} < \gamma < \gamma^{T2}$ is also described by Figure 6.4(c) for $\theta = 20$, which confirms that the low equilibrium position is locally unstable and shows that, if $\gamma < \bar{\gamma}$, increases in γ generate many period-doubling bifurcations in the economic system up to chaos. On the contrary, if $\gamma > \bar{\gamma}$, increases in γ reduce the period and the size of fluctuations, contributing to rid the economic system of chaotic behaviour. As to the high equilibrium position, for $\gamma^{F1} \leq \gamma < \gamma^{T1}$ the economic system moves from a 2-cycle situation to a unique equilibrium position. For $\gamma \geq \gamma^{F2}$ the opposite occurs and the economic system returns to a 2-cycle situation.

To sum up, analysis of the effects of changes in what has been called 'productive' government expenditure is more ambiguous than that of the effects of 'unproductive' expenditure. It shows that, for e_H, increases in government expenditure tend to have a positive influence on the stability and controllability of the economic system when they generate an increase in after-tax profits. This influence is however negative when government expenditure approaches the level that maximises after-tax profits. By contrast, when they reduce after-tax profits the influence on the stability and controllability of the economic system is positive up to a level, beyond which it becomes negative. As to the influence on growth, it is negative when after-tax profits increase and positive when they decrease.

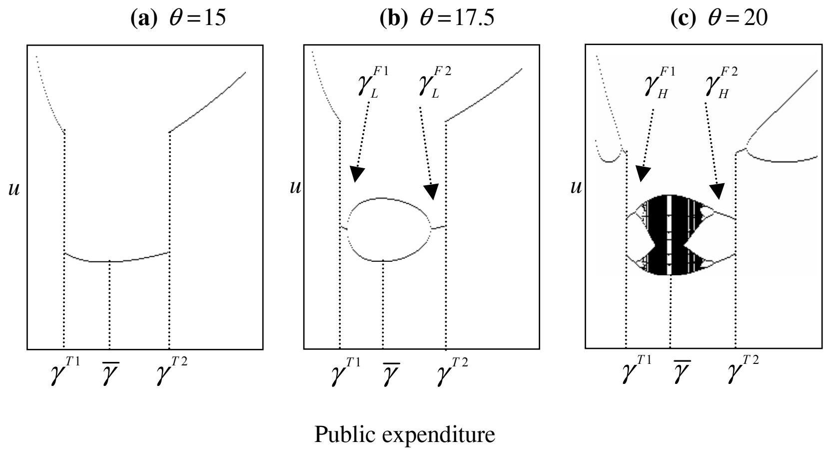

Figure 6.4. Kaleckian interpretation: bifurcation diagrams showing the impact of 'productive' public expenditure γ on the long-term behaviour of the state variable u for different values of the adjustment speed θ.

6.4. THE CLASSICAL–HARRODIAN INTERPRETATION

6.4.1. Equilibrium

The results of the Classical–Harrodian case may differ from those of the Kaleckian case because the rate of growth and the degree of capital utilisation may move in opposite directions when government expenditure changes. The former case interprets the expected growth rate of demand as the warranted rate of growth, i.e. $\alpha = \tilde{g} \equiv s_\pi \pi a(1-\gamma)\tilde{u}$. The equilibrium solutions are

$$u^* = \frac{\tilde{g} + \phi(u^*)}{s_\pi \pi a(1-\gamma)} \quad \text{and} \quad g^* = \tilde{g} + \phi(u^*) \tag{6.19}$$

Commendatore et al. (2007; 2009) present the diagram reported in Figure 6.5, which refers to the case of 'unproductive' expenditure and shows that, according to the value attributed to γ, there can be either one or three equilibrium positions, $e_L \equiv (u_L^*, g_L^*)$, $\tilde{e} \equiv (\tilde{u}, \tilde{g})$ and $e_H \equiv (u_H^*, g_H^*)$.[12] In Figure

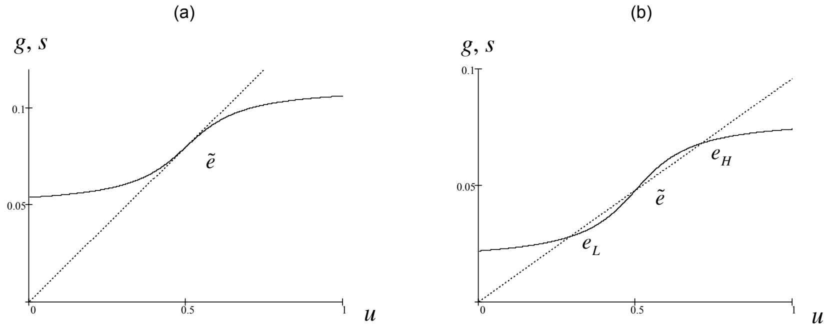

Figure 6.5. Classical-Harrodian interpretation: saving and investment functions for different values of public expenditure. (a) only the Harrodian warranted growth equilibrium exists; (b) three equilibria are present (low, Harrodian and high)

6.5(a) there is one equilibrium position, $\tilde{e}$, which is globally stable because condition (6.15) holds. In Figure 6.5(b) the increase in γ generates a pitchfork bifurcation and the appearance of two new equilibrium positions, e_L and e_H, which are symmetrical with respect to $\tilde{e}$. The equilibrium positions e_L and e_H are locally stable, while $\tilde{e}$ is unstable because condition (6.15) is violated. Notice that increases in 'productive' government expenditure, which raise after-tax profits, could reverse the direction of the bifurcation process.

Commendatore et al. (2007; 2009) also show that there are no hysteresis effects and that increases in 'unproductive' government expenditure lower the value of u_L and raise that of u_H, while increases in 'productive' expenditure, which positively affect after-tax profits, raise the value of u_L and lower that of u_H.

6.4.2. Dynamic Analysis

Let us now present the dynamic analysis, investigating how changes in θ affect the long-term behaviour of the economy. Subsequently we explore the influence of changes in government expenditure on the local stability of equilibrium positions.

The difference equation (6.12) is revised as follows for the Classical–Harrodian case:

$$\psi(u) = u + \theta\left[\phi(u) - s_\pi \pi a(1-\gamma)(u - \tilde{u})\right] \tag{6.20}$$

This map determines the long-term evolution of the state variable *u*. A crucial property of this map is that it is symmetric around the equilibrium $\psi(\tilde{u}) = \tilde{u}$.

The equilibrium positions u_L and u_H are stable for

$$\theta < \theta^F \equiv \frac{2}{s_\pi \pi a(1-\gamma) - \phi'(u^*)}$$

Due to the symmetry of map (6.12), the bifurcation point θ^F is the same for both equilibrium positions since $\phi'(u_L^*) = \phi'(u_H^*)$. When θ increases both equilibrium positions lose stability through a flip bifurcation and two locally stable 2-cycles emerge.[13] Further increases in θ generate first the appearance of two symmetric attractors of various periodicity and then the emergence of chaotic attractors. The attractors, located around the low and high equilibrium positions, enjoy the same periodicity. As θ increases, their basins become more intertwined up to when, for a sufficiently large θ, a global bifurcation takes place and the two basins merge.

Figure 6.6 presents two bifurcation diagrams which describe the influence of changes in θ – varied within the interval $35 \le \theta \le 55$ – on the long-term behaviour of the state variable *u* and on the stability properties of the low and high equilibrium positions.[14] Both diagrams confirm that a period-doubling

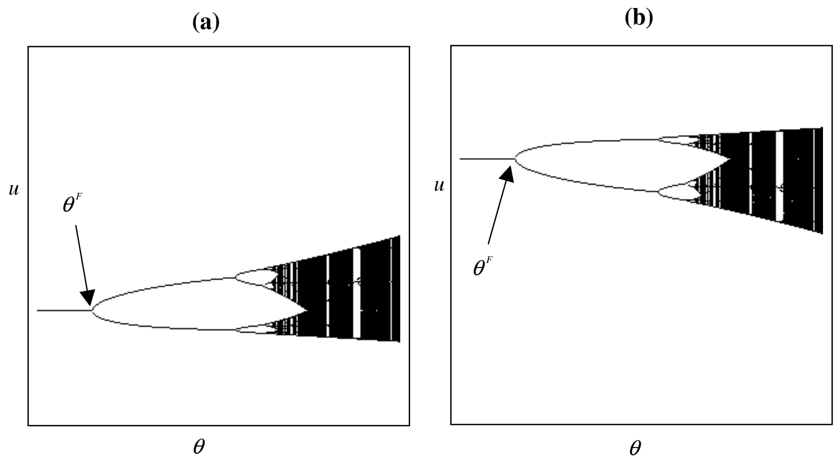

Figure 6.6. Classical-Harrodian interpretation: bifurcation diagrams showing the impact of the adjustment speed θ on the long-term behaviour of the state variable u for different initial conditions. (a) the initial condition belongs to the basin of attraction of the low equilibrium; (b) the initial condition belongs to the basin of attraction of the high equilibrium

route to complex behaviour occurs simultaneously for the low and high equilibrium positions. A multiplicity of symmetric attractors is created whose stability properties also follow a symmetric pattern.

Let us now examine the impact of a change in γ on the dynamic properties of the map $\psi(u)$. Consider first the case of a change in 'unproductive' government expenditure and its effects on the bifurcation point θ^F.

$$\frac{d\theta^F}{d\gamma} = \theta^F \frac{s_\pi \pi a + \phi''(u^*) du^*/d\gamma}{s_\pi \pi a(1-\gamma) - \phi'(u^*)} .$$

The sign of $d\theta^F/d\gamma$ is indeterminate because for the low equilibrium position $\phi''(u_L^*) > 0$ and $du_L^*/d\gamma < 0$ and for the high equilibrium position $\phi''(u_H^*) < 0$ and $du_H^*/d\gamma > 0$. Thus the effect of a change in government expenditure on the local stability is not unambiguous.

When government expenditure is 'productive', the effects of its changes on the bifurcation point θ^F are described by the following derivative:

$$\frac{d\theta^F}{d\gamma} \equiv \theta^F \frac{s_\pi a\left[\pi(\gamma) - \pi'(\gamma)(1-\gamma)\right] + \phi''(u^*)\dfrac{du^*}{d\gamma}}{s_\pi \pi(\gamma) a(1-\gamma) - \phi'(u^*)}$$

The sign of this derivative is also indeterminate. When after-tax profits are increased by the rise in government expenditure, i.e. $\pi(\gamma) - \pi'(\gamma)(1-\gamma) < 0$, for the low equilibrium $\phi''(u_L^*) > 0$ and $du_L^*/d\gamma > 0$, whereas for the high equilibrium $\phi''(u_H^*) < 0$ and $du_H^*/d\gamma < 0$. When instead $\pi(\gamma) - \pi'(\gamma)(1-\gamma) > 0$, for the low equilibrium $\phi''(u_L^*) > 0$ and $du_L^*/d\gamma < 0$, whereas for the high equilibrium $\phi''(u_H^*) < 0$ and $du_H^*/d\gamma > 0$.

To explore further the effects of government expenditure on the local stability of the economic system some simulations were made.

Figure 6.7 presents the bifurcation diagrams, 6.7(a) and 6.7(b), showing the impact of $0 \le \gamma \le 0.5$ on the long-term behaviour of the degree of capacity utilisation for $\theta = 40$.[15] Figure 6.7(a) deals with the case of 'unproductive' government expenditure and Figure 6.7(b) with the case of 'productive' expenditure.

Figure 6.7(a) shows two flip bifurcations, occurring at $\gamma = \gamma^{F1}$ and $\gamma = \gamma^{F2}$, for the high equilibrium position. By symmetry two other bifurcations occur at the same values of γ for the low equilibrium position. For $0 \le \gamma < \gamma^{F1}$ these equilibrium positions are stable. For $\gamma^{F1} < \gamma < \gamma^{F2}$ two locally attracting 2-cycles emerge. For $\gamma^{F2} < \gamma \le 0.5$ the couple of 2-cycles disappear and the low and high equilibrium positions become locally attracting once again.

These results mean that, taking the bifurcation points as reference, small and big sizes of the government sector have stabilising effects on the economic system. Moreover, increases in 'unproductive' government expenditure have a positive effect on u_H^* and, by symmetry, a negative effect on u_L^*. With respect to the Kaleckian case both 'small' and 'big' governments generate stabilising effects on the economy. Moreover, owing to the negative effect on u_L^*, increases in the size of the government sector may produce depressing effects on the economy.

Figure 6.7(b) shows four flip bifurcations for the high equilibrium position. By symmetry, four other bifurcations occur at the same values of γ for the low equilibrium position. For $0 \leq \gamma < \gamma^{F1}$ the equilibrium positions are stable. For $\gamma^{F1} < \gamma < \gamma^{F2}$ two locally attracting 2-cycles emerge. For $\gamma^{F2} < \gamma < \gamma^{F3}$ the equilibrium positions are stable again. An attracting 2-cycle emerges again for $\gamma^{F3} < \gamma < \gamma^{F4}$. For $\gamma^{F4} < \gamma \leq 0.5$ the pair of 2-cycles disappear and the low and high equilibrium positions become locally attracting once again.

These results mean that increases in 'productive' government expenditure have stabilising effects on the economic system when the government sector is 'big' enough to overtake γ^{F4} and when it approaches the value that maximises after-tax profits, $\bar{\gamma}$. By contrast, reductions in 'productive'

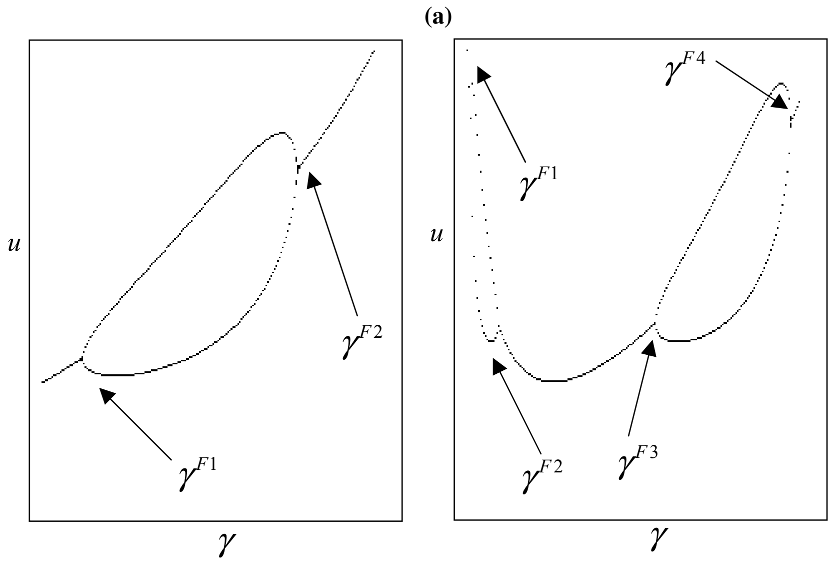

Figure 6.7. Classical-Harrodian interpretation: bifurcation diagrams showing the impact of public expenditure γ on the long-term behaviour of the state variable u. (a) 'unproductive' public expenditure; (b) 'productive' public expenditure.

government expenditure have stabilising effects on the economic system when the size of the government sector is 'small', i.e. it is less than γ^{F1}, and when, approaching $\bar{\gamma}$, it becomes less than γ^{F3}. Moreover, increases in 'productive' government expenditure have a negative effect on u_H^* and, by symmetry, a positive effect on u_L^* when after-tax profits rise. On the contrary, when after-tax profits diminish, increases in 'productive' government expenditure have a positive effect on u_H^* and, by symmetry, a negative effect on u_L^*.

To sum up, the results of Classical–Harrodian analysis of changes in 'productive' government expenditure differ from those of the Kaleckian case because, when the size of the government sector approaches the level that maximises after-tax profits, the economic system converges to a stable equilibrium position whereas in the Kaleckian case it shows a chaotic behaviour.

6.5. CONCLUSIONS

Our analysis shows that the introduction of a discrete time framework and a nonlinear investment function generates a wide range of complex phenomena which are absent in post-Keynesian linear models. The long-term behaviour of the economic system depends on the speed at which firms adjust capacity utilisation in reaction to excess demand. If the adjustment is slow, the economy converges to a stable equilibrium position. In the opposite case, a period-doubling sequence to complex behaviour can occur both in the Kaleckian and Classical–Harrodian analysis.

What we called 'unproductive' and 'productive' government expenditures produce different effects on equilibrium positions and their local stability in the Kaleckian and the Classical–Harrodian interpretations.

In the Kaleckian interpretation, increases in 'unproductive' government expenditure have a stabilising effect on the low equilibrium position. They also have stabilising effects on the high equilibrium position as long as the size of the government sector is sufficiently large. The existence of a 'big government' favours a high degree of capital utilisation and high rates of growth.

Increases in 'productive' expenditure have destabilising effects on the low equilibrium position when after-tax profits rise and stabilising effects when after-tax profits diminish. The high equilibrium position is instead stabilised by the increase in 'productive' expenditure when after-tax profits rise and destabilised when after–tax profits diminish (see Commendatore et al., 2007; 2009).

In the Classical–Harrodian interpretation increases in ‘unproductive’ government expenditure stabilise the economic system when the government sector is sufficiently large. The existence of a ‘big government’, however, pushes the degree of capital utilisation further above its normal level for the high equilibrium position and further below it for the low equilibrium position.

The effects of increases in ‘productive’ expenditure are stabilising when the dimension of the government sector approaches the value that maximises after-tax profits and when it is sufficiently large. On the contrary, reductions in ‘productive’ expenditure have stabilising effects when the government sector is sufficiently ‘small’ and when it approaches the value that maximises after-tax profits. Moreover, unlike the Kaleckian case that shows chaotic behaviour, the economic system converges to a stable equilibrium position when the dimension of the government sector approaches the level that maximises after-tax profits. Finally, the effect of ‘productive’ expenditure on the degree of capacity utilisation, for both the low and high equilibrium, is ambiguous depending on the behaviour of after-tax profits (see Commendatore et al., 2007; 2009).

NOTES

1. Keynes (1924) underlined the crucial role of the composition of government expenditure since he first called for the use of fiscal policy to stimulate the economy, while Kaldor (1958, pp. 136–7; 1966; 1967; and 1971) pointed out that the composition of government expenditure has important effects on long-run growth.
2. The definition of ‘unproductive’ and ‘productive’ government expenditure is given in Section 6.2.
3. Mastromatteo (2009) reports that Minsky, following a Kaleckian theory of growth and distribution, argues for the positive effects of the existence of a ‘big government’ on the growth and controllability of the system.
4. For the sake of simplicity we do not incorporate the rate of profit into the investment function. Our results would not be substantially modified if we change this equation to include the profit rate.
5. We assume $\alpha + \phi(0) > 0$, that is the long-run expected growth of demand α is always high enough to induce positive investments even in correspondence of a low capacity utilisation in the current period (this assumption is standard in the Kaleckian literature: see Lavoie, 1996; see also Kaldor, 1940). Hence $u^* > 0$ and $g^* > 0$.
6. Figure 6.1 assumes that γ takes the following values: (a) $\gamma = 0$, (b) $\gamma = 0.12$ and (c) $\gamma = 0.24$. The other parameters are given the following values: $\tilde{u} = 0.5$, , $\alpha = 0.07$, $a = 0.5$, $\lambda = 0.75$, $s_\pi = 0.8$ and $w_0 = 0.6$ As an explicit form for the nonlinear component of the investment function we choose $\phi(u) = \beta_1 \arctan[\beta_2(u - \tilde{u})]$, where $\beta_1 = 0.015$ and $\beta_2 = 15$; and as an explicit form for the average labour productivity function we choose $b(\gamma) = b_0 + b_1 \arctan(b_2\gamma)$, where $b_0 = 1$, $b_1 = 0$ and $b_2 = 7.5$. Notice that the choice of this parameter constellation corresponds to the assumption that public expenditure does not affect the labour input coefficient $b' = 0$.

7. This is a typical result of the use of non-monotonic maps (see Alligood et al., 1997). A thorough analysis of the dynamical properties of map (6.17) is presented in Commendatore et al. (2007).
8. See previous footnote 6.
9. The technical details of this analysis and the corresponding diagrams are given in Commendatore et al. (2007).
10. To plot Figure 6.4, for the other parameters, we set the same configuration as Figure 6.1; and $u_0 = 0.51$ as initial condition.
11. To plot Figure 6.4 we used, for the other parameters, $\tilde{u} = 0.5$, $a = 0.5$, $s_\pi = 0.8$, $\beta_1 = 0.02$, $\beta_2 = 15$, $b_0 = 1$, $b_1 = 1.15$, $b_2 = 7.5$, $\lambda = 0.6$ and $w_0 = 0.72$. Moreover, we set as initial condition $u_0 = 0.51$.
12. To plot Figure 6.6, we used the following parameter constellation: $\tilde{u} = 0.5$, $a = 0.5$, $s_\pi = 0.8$, $\lambda = 0.75$, $w_0 = 0.6$, $\beta_1 = 0.02$, $\beta_2 = 7.5$, $b_0 = 1$, $b_1 = 0$, $b_2 = 7.5$, $\gamma = 0$ for panel (a) and $\gamma = 0.4$ for panel (b). As for Figure 6.1, this parameter configuration corresponds to the assumption $b' = 0$.
13. For a detailed description of this process, see Commendatore et al. (2007).
14. The diagram uses initial values (a) $u_0 = 0.49$ (belonging to the basin of attraction of the low equilibrium) and (b) $u_0 = 0.51$ (belonging to the basin of attraction of the high equilibrium) and the same parameter constellation as Figure 6.5(b).
15. To plot Figure 6.7(a) we used the following parameter constellation: $\tilde{u} = 0.5$, $a = 0.5$, $s_\pi = 0.8$, $\lambda = 0.75$, $w_0 = 0.72$, $\beta_1 = 0.02$, $\beta_2 = 7.5$, $b_0 = 1$, $b_1 = 0$, $b_2 = 7.5$. Moreover, we set $u_0 = 0.51$ as initial condition. To plot Figure 6.7(b) we used the following parameter constellation: $\tilde{u} = 0.5$, $a = 0.5$, $s_\pi = 0.8$, $\beta_1 = 0.02$, $\beta_2 = 7.5$, $b_0 = 0.5$, $b_1 = 1.12$, $b_2 = 7.5$, $\lambda = 0.7$ and $w_0 = 0.72$. Moreover, we set as initial condition $u_0 = 0.51$.

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